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Analysis of SIR Model Dynamics with Vaccination Strategies and Graph-Based Ethnomathematical Representation

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Abstract: Tuberculosis (TB) ranks Indonesia second globally in terms of the highest case burden. This study aims to develop a modified Susceptible-Infected-Recovered (SIR) model with vaccination strategies to evaluate and optimise national TB control efforts. Model parameters were adjusted based on the latest epidemiological data, including under-reporting rates and case incidence in Daerah Istimewa Yogyakarta (DIY) from the 2023 Indonesian Tuberculosis Inventory Study Report. Mathematical analysis focused on determining key model parameters and equilibrium stability to identify the conditions that define the boundary between disease sustainability and potential elimination. Numerical simulations compared in detail the effectiveness of constant and periodic vaccination, highlighting the optimal scheme capable of reducing the number of infected cases (I) more quickly. The uniqueness of this research lies in the integration of science and culture through ethnomathematics, where the dynamic patterns of the SIR phase space model are processed into distinctive and meaningful batik motifs. This research supports the development of more effective TB vaccination policies and examines the dynamic relationship between epidemiology and the richness of Indonesian visual arts.

Keywords: Ethnomathematics, Mathematical Modelling, SIR Model, Tuberculosis, Vaccination.

Introduction

Advances in science and technology in the field of medicine have encouraged experts to continue researching various types of diseases, one of which is infectious diseases. Infectious diseases are diseases caused by bacteria, viruses, or parasites that can be transmitted to other people. Transmission can occur through various media, such as air, syringes, blood transfusions, eating and drinking utensils, and others. Infectious diseases arise from the interaction of various factors that are interrelated and influence each other.

The emergence and re-emergence of diseases has led to increased attention to infectious diseases. This condition is called Re-emerging Infectious Diseases. The mode of transmission from infected people to susceptible people is already well known. However, the process of disease spread in the community is very complex, making it difficult to understand the dynamics of large-scale disease spread without the formal structure of mathematical models. Mathematical modeling in the spread of infectious diseases has become an important part of the epidemiological policy decision-making process in various developed countries, such as the United Kingdom, the Netherlands, Canada, and the United States. Therefore, the mathematical modeling approach is very important for decision-making regarding infectious disease control programs.

In observing the spread of infectious diseases mathematically, epidemiological mathematical models are used, one of which is the SIR (Susceptible Infected Recovered) model discovered by (Kermack & McKndrick, 1927). In their research, they linked this to determining the threshold value or basic reproduction number (R_0). In the

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dynamics of mathematical epidemiological models, we will see how the model behaves around the equilibrium point. This behavior is analyzed through stability analysis determined from the eigen value of the Jacobian matrix of each equilibrium point.

This study discusses the dynamics of the SIR model with a vaccination strategy. Vaccination is the administration of antigens from viruses or bacteria that can stimulate the body's immune system (antibodies). The vaccination strategy is carried out in two ways, namely constant vaccination and periodic vaccination of individuals who are susceptible to infection. Numerical simulations were performed using Mathlab software to confirm the analytical results using parameters from the case of tuberculosis (TB) in Indonesia in 2023

Method And Model

Modified SIR Model with Vaccination

The epidemiological model used in this study is a modification of the standard SIR model (Susceptible-Infected-Recovered). This model divides the total population (N) into three main segments that depend on time t, there are:

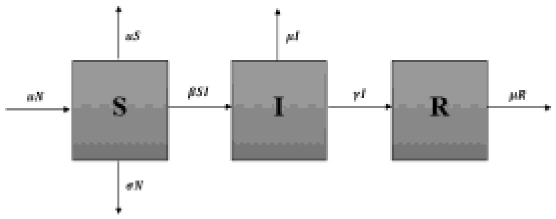
$S(t)$ = Susceptible Population, individuals who can be infected with tuberculosis.

$I(t)$ = Infected Population, individuals suffering from active tuberculosis and transmitting the disease.

$R(t)$ = Recovered Population.

The total population is assumed as follows.

$$N(t) = S(t) + I(t) + R(t).$$



Assume that vaccination is applied to the Susceptible Population (S) segment.

The SIR model with constant vaccination can be expressed as a system of ordinary differential equations (ODEs) as follows.

$$f_1 = \frac{ds}{dt} = (\mu - \sigma)N - \beta SI - \mu S$$
$$f_2 = \frac{dI}{dt} = \beta SI - \mu I - \gamma I$$
$$f_3 = \frac{dR}{dt} = \gamma I - \mu R$$

Description:

$\frac{ds}{dt}$ = Rate of change of the Susceptible Population

$\frac{dI}{dt}$ = Rate of change of the Infected Population

$\frac{dR}{dt}$ = Rate of change in the Recovered Population

with the parameters used being:

β = Birth & death rates

μ = Recovery rate

γ = Transmission rate

σ = Vaccination rate

Equilibrium point

In this study, stability (equilibrium) analysis will be discussed using the SIR model with constant vaccination. There are two equilibrium points that describe two conditions, namely disease-free (EP_0) and endemic (EP^*) which indicate the condition of infected individuals in the population.

Disease-Free Equilibrium point ($\boldsymbol{EP_0}$)

The disease-free equilibrium point occurs when there are no infected individuals ($I = 0$), then:

$$\frac{ds}{dt} = (\mu - \sigma)N - \mu S = 0$$
$$\mu S = (\mu - \sigma)N$$
$$S_0 = \frac{(\mu - \sigma)N}{\mu};$$
$$\frac{dI}{dt} = 0 = I_0;$$
$$\frac{dR}{dt} = \gamma I - \mu R = 0$$
$$\mu R = \gamma I$$
$$R_0 = 0.$$

then we have:

$$EP_0 = (S_0, I_0, R_0)$$
$$= \left(\frac{(\mu - \sigma)N}{\mu}, 0,0\right)$$

Endemic Disease Equilibrium Point

The disease-free equilibrium point occurs when there are no infected individuals ($I > 0$), then:

$$\frac{dR}{dt} = \gamma I - \mu R = 0$$
$$\mu R = \gamma I$$
$$R^* = \frac{\gamma I^*}{\mu};$$
$$\frac{dI}{dt} = (\beta S - \mu - \gamma)I = 0$$
$$\beta S = \mu + \gamma$$
$$S^* = \frac{\mu + \gamma}{\beta};$$
$$\frac{ds}{dt} = (\mu - \sigma)N - \beta SI - \mu S = 0$$
$$(\mu - \sigma)N - \beta\left(\frac{\mu + \gamma}{\beta}\right)I - \mu\left(\frac{\mu + \gamma}{\beta}\right) = 0$$
$$(\mu - \sigma)N - \mu\left(\frac{\mu + \gamma}{\beta}\right) = \beta\left(\frac{\mu + \gamma}{\beta}\right)I$$
$$\frac{(\mu - \sigma)N - \mu(\mu + \gamma)}{\beta(\mu + \gamma)} = I^*$$

then we have:

$$EP^* = (S^*, I^*, R^*) = \left(\frac{\mu + \gamma}{\beta}, \frac{(\mu - \sigma)N - \mu(\mu + \gamma)}{\beta(\mu + \gamma)}, \frac{\gamma I^*}{\mu}\right)$$

From the above system of equations, we obtain:

$$\frac{\partial f_1}{\partial S} = -\beta I - \mu, \quad \frac{\partial f_2}{\partial S} = -\beta I, \quad \frac{\partial f_3}{\partial S} = 0$$
$$\frac{\partial f_1}{\partial I} = -\beta S, \quad \frac{\partial f_2}{\partial I} = -\beta S - \mu - \gamma, \quad \frac{\partial f_3}{\partial I} = \gamma$$
$$\frac{\partial f_1}{\partial R} = 0, \quad \frac{\partial f_2}{\partial R} = 0, \quad \frac{\partial f_3}{\partial R} = -\mu$$

Thus the Jacobian matrix can be formed like this:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial R} \end{bmatrix} = \begin{bmatrix} -\beta I - \mu & -\beta S & 0 \\ -\beta I & -\beta S - \mu - \gamma & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

Results And Discussion

Stability Analysis

After found the equilibrium points and their Jacobian Matrix, the stability at these points is analyzed.

Stability at the point EP_0

The Jacobian Matrix at the equilibrium point EP_0:

$$JEP_0 = \begin{bmatrix} -\mu & \frac{-\beta(\mu - \sigma)N}{\mu} & 0 \\ 0 & \frac{-\beta(\mu - \sigma)N}{\mu} - \mu - \gamma & 0 \\ 0 & \gamma & -\mu \end{bmatrix} = \begin{bmatrix} -\mu & \frac{-\beta(\mu - \sigma)N}{\mu} & 0 \\ 0 & \frac{-\beta(\mu - \sigma)N - \mu(\mu + \gamma)}{\mu} & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

The following characteristic equation can be obtained:

$$|JEP_0 - \lambda I| = 0$$
$$\begin{vmatrix} -\mu - \lambda & \frac{-\beta(\mu - \sigma)N}{\mu} & 0 \\ 0 & \frac{-\beta(\mu - \sigma)N - \mu(\mu + \gamma)}{\mu} - \lambda & 0 \\ 0 & \gamma & -\mu - \lambda \end{vmatrix} = 0$$

Therefore:

$$(-\mu - \lambda)\begin{vmatrix} \frac{-\beta(\mu - \sigma)N - \mu(\mu + \gamma)}{\mu} - \lambda & 0 \\ \gamma & -\mu - \lambda \end{vmatrix} = 0$$
$$(-\mu - \lambda)\left(\left(\frac{-\beta(\mu - \sigma)N - \mu(\mu + \gamma)}{\mu} - \lambda\right)(-\mu - \lambda) - 0 \cdot \gamma\right) = 0$$

We obtain:

$$\lambda_1 = \lambda_2 = -\mu\,;\ \lambda_3 = \frac{\beta(\mu - \sigma)N - \mu(\mu + \gamma)}{\mu}$$

Stability at point EP^*

The Jacobian Matrix at the equilibrium point EP^* are

$$JEP^* = \begin{bmatrix} -\beta\left(\frac{(\mu-\sigma)N-\mu(\mu+\gamma)}{\beta(\mu+\gamma)}\right)-\mu & -\beta\left(\frac{\mu+\gamma}{\beta}\right) & 0 \\ \beta\left(\frac{(\mu-\sigma)N-\mu(\mu+\gamma)}{\beta(\mu+\gamma)}\right) & \beta\left(\frac{\mu+\gamma}{\beta}\right)-\mu-\gamma & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-\beta\mu N+\sigma\beta N}{\beta(\mu+\gamma)}+\frac{\beta\mu(\mu+\gamma)}{\beta(\mu+\gamma)} & -\beta\left(\frac{\mu+\gamma}{\beta}\right) & 0 \\ \frac{\beta\mu N+\sigma\beta N}{\beta(\mu+\gamma)}+\frac{\beta\mu(\mu+\gamma)}{\beta(\mu+\gamma)} & \beta\left(\frac{\mu+\gamma}{\beta}\right)-\mu-\gamma & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

$$JEP^* = \begin{bmatrix} \frac{-(\mu-\sigma)N\beta}{(\mu+\gamma)} & -(\mu+\gamma) & 0 \\ \frac{(\mu+\sigma)N\beta-\mu(\mu+\gamma)}{(\mu+\gamma)} & 0 & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

Thus, the characteristic equation of the JEP^* is obtained as follows:

$$|JEP^* - \lambda I| = 0$$

$$\begin{vmatrix} \frac{-(\mu-\sigma)N\beta}{(\mu+\gamma)}-\lambda & -(\mu+\gamma) & 0 \\ \frac{(\mu+\sigma)N\beta-\mu(\mu+\gamma)}{(\mu+\gamma)} & -\lambda & 0 \\ 0 & \gamma & -\mu-\lambda \end{vmatrix} = 0$$

Therefore,

$$-(\mu+\gamma)\begin{vmatrix} \frac{-(\mu-\sigma)N\beta}{(\mu+\gamma)}-\lambda & -(\mu+\gamma) \\ \frac{(\mu+\sigma)N\beta-\mu(\mu+\gamma)}{(\mu+\gamma)} & -\lambda \end{vmatrix} = 0$$

$$-(\mu+\gamma)\left(\left(\frac{-(\mu-\sigma)N\beta}{(\mu+\gamma)}-\lambda\right)(-\lambda) - \left(-(\mu+\gamma)\right)\left(\frac{(\mu+\sigma)N\beta-\mu(\mu+\gamma)}{(\mu+\gamma)}\right)\right) = 0$$

$$-(\mu+\gamma)\left(\lambda\left(\lambda+\frac{(\mu-\sigma)N\beta}{(\mu+\gamma)}\right) + (\mu+\gamma)\left(\frac{(\mu+\sigma)N\beta-\mu(\mu+\gamma)}{(\mu+\gamma)}\right)\right) = 0$$

Assume that

$$x = \frac{(\mu-\sigma)N\beta}{(\mu+\gamma)}$$
$$y = (\mu+\sigma)N\beta - \mu(\mu+\gamma)$$

Thus, the characteristic equation is obtained as follows:

$$-(\mu+\gamma)(\lambda^2 + x\lambda + y) = 0$$

We obtain:

$$\lambda_1 = -\mu$$
$$\lambda_{2,3} = \frac{-x+\sqrt{x^2-4y}}{2}$$

$$= \frac{-\frac{(\mu-\sigma)N\beta}{(\mu+\gamma)}\sqrt{\left(\frac{(\mu-\sigma)N\beta}{(\mu+\gamma)}\right)^2 - 4\left((\mu+\sigma)N\beta-\mu(\mu+\gamma)\right)}}{2}$$

$$= \frac{-(\mu-\sigma)N\beta}{2(\mu+\gamma)} \pm \frac{1}{2}\sqrt{\left(\frac{(\mu-\sigma)N\beta}{(\mu+\gamma)}\right)^2 - 4\left((\mu+\sigma)N\beta-\mu(\mu+\gamma)\right)}$$

$$\lambda_{2,3} = \frac{1}{2}\left(\frac{-(\mu-\sigma)N\beta}{2(\mu+\gamma)} + \sqrt{\left(\frac{(\mu-\sigma)N\beta}{(\mu+\gamma)}\right)^2 - 4\left((\mu+\sigma)N\beta-\mu(\mu+\gamma)\right)}\right)$$

Numerical Simulaation:

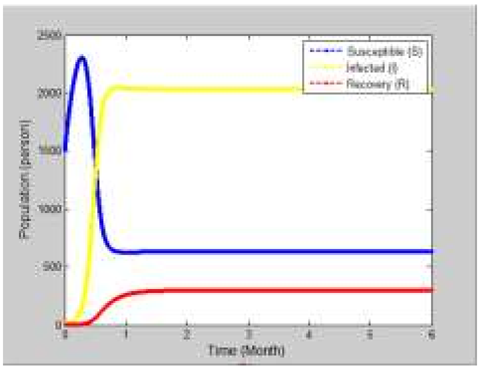


Figure 1. Numerical Simulation

In this article, the parameter values for the TBC case in the Special Region o Yogyakarta (Daerah Istimewa Yogyakarta (DIY)) in 2023 are used, obtained from tuberculosis (TBC_ data in Indonesia for the years 2023. The values of each parameters are used: $N = 3736; \mu = 3.85; \gamma = 0.56; \beta = 0.003; \sigma = 0.8$ with initial values $S(0) = 1500; I(0) = 10; R(0) = 5$.

Vaccination at constant rate $\sigma = 0.8$, this reduces the number of susceptible individuals an suppresses the rate of infection. However, infected individuals still remain in the population. The figure also shows that the higher the vaccination rate, the lower the number o infected individuals and vice versa.

Application on Ethnomathematics (Batik Motifs)

From the simulation above, the resulting graphic can be implemented and utilized as a batik motif, as shown in the experiment below. The batik motif obtained is a repeating spiral pattern with long waves. This batik motif is combined with the Kawung Batik pattern.

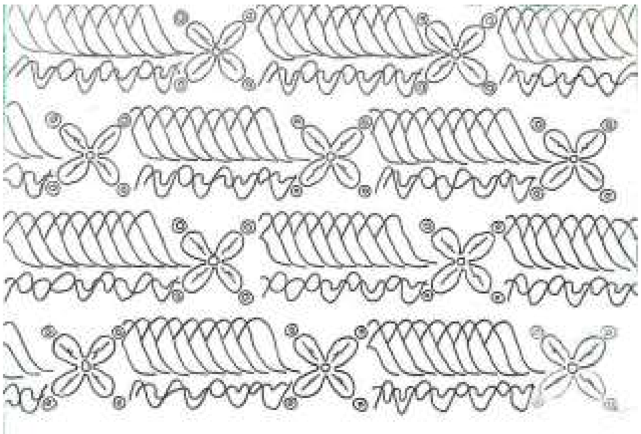


Figure 1. Motif Batik of Simulation Graphic

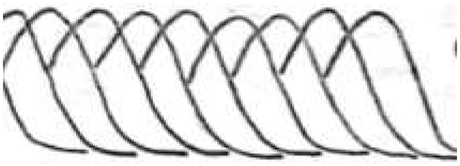


Figure 2. Repeating Spiral Pattern

Figure 3. Long Waves Pattern

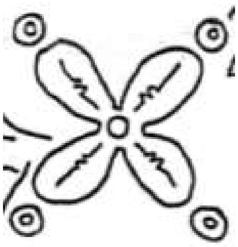


Figure 4. Kawung Batik

All these batik motifs contain philosophical meanings in terms of art, biology, and mathematics.

1. Philosophical Means

This batik motif contains a philosophy about the journey of human life, which is always moving in an upward and downward rhythm. The repeating spiral motif symbolizes a life journey that continues to move and evolve. This pattern illustrates that each phase of life has its own rhythm and cycle, but all of them lead humans towards growth and harmony. Meanwhile, the long wave motif symbolizes the dynamics of life, which is full of ups and downs. This pattern illustrates the human ability to adapt and find balance amid constant change.

The Kawung batik motif symbolizes purity, balance, and self-control. Its symmetrical oval arrangement reflects inner harmony and encourages a simple, fair, and steadfast life in the face of change.

2. Biological Means

This batik motif is inspired by patterns that appear in SIR model simulations with vaccination, which is a biological model that describes the dynamics of infectious disease spread in a population.

The repeating spiral lines represent changes in the number of individuals in the susceptible (S), infected (I), and recovered (R) groups, with vaccination shifting the pattern so that the peak of infection is lower. Meanwhile, the wavy lines along the motif symbolize fluctuations in the rate of transmission over time, emphasizing that epidemics are dynamic and constantly changing in response to health interventions. Overall, this batik motif transforms the mathematical form of the SIR model with vaccination into a harmonious visual, emphasizing the importance of vaccination and interactions between individuals in maintaining public health balance.

3. Mathematical Means

The mathematical meaning of this batik shows the oscillatory change in population over time around the equilibrium point. The repeating waves and curves in the motif are a visualization of the solution to a system of differential equations that describes the interaction between susceptible, infected, and recovered populations and the effect of vaccination.

Stability analysis through the eigenvalues of the Jacobian shows that changes in parameters such as transmission rate or vaccine effectiveness can cause oscillations around the equilibrium state. The repeating patterns in the batik reflect the ups and downs of the population in the model, which moves through a repeating trajectory before finally approaching a stable state. Overall, this batik represents order and dynamics in mathematical systems, where quantitative interactions between variables produce patterns of oscillation, stability, and changes over time.

Conclusions

The results of the analysis conducted on the 2023 tuberculosis (TB) case in the Special Region of Yogyakarta (DIY) show that vaccination has a significant effect on changes in system dynamics, with increased vaccination rates reducing the number of infected individuals and pushing the system towards disease-free conditions. These findings are consistent with numerical simulations using tuberculosis (TB) case parameters in West Sumatra Province in 2018, which show that vaccination interventions can shift the peak of infection and reduce the intensity of disease spread.

The mathematical dynamics generated from the simulation were then utilized in the batik motif design. Repeating spiral patterns and long waves were taken from graphic shapes resembling the oscillation patterns of the SIR model solution, particularly when the system moves closer to its stability point. These patterns illustrate the rise and fall of susceptible, infected, and recovered populations over time. In addition, the kawung batik motif was added as a cultural element that enriches the design.

Overall, this study concludes that the SIR model with vaccination strategies not only plays an important role in understanding the mechanisms of disease spread, but can also be used as a basis for developing batik artworks that have philosophical, biological, and mathematical value. This integration between science and art shows that mathematical modeling can transcend its function as an analytical tool, becoming a source of meaningful visual inspiration and a creative educational tool for the community.

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