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# Geometric Thinking Ability Based on Van Hiele's Theory in Ethnomathematics-Based Conicongruence Learning at Museum *Benteng Vredeburg*

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**Abstract:** This study aims to determine the effectiveness of learning and analyse the improvement of geometric thinking levels according to Van Hiele's theory. As well as how students' experience of learning ethnomathematics-based congruence at Museum Benteng Vredeburg. The ethnomathematics approach is used to integrate local cultural concepts into mathematics learning, in order to improve students' geometric thinking skills. This study used quantitative and qualitative approaches involving junior high school students as research subjects. Data were collected through geometry thinking ability tests, interviews, and observations during the learning process. Analysis was conducted to identify students' geometry thinking level according to the five stages in Van Hiele's Theory, namely visualisation, analysis, informal deduction, deduction, and rigor. The results showed that the ethnomathematics-based learning approach was effectively implemented and was able to facilitate students in reaching higher stages of geometry thinking. The use of cultural elements, such as the building structure in Museum Benteng Vredeburg, provides a relevant concrete context and increases the level of geometry thinking. This research contributes to the development of culture-based mathematics learning design, especially to improve students' geometry thinking skills.

**Keywords:** geometry thinking skills, Van Hiele Theory, congruence, ethnomathematics, *Museum Benteng Vredeburg*.

## Introduction

Geometry learning has an important role in developing students' critical thinking, problem solving, and deductive reasoning skills (Tursynkulova et al., 2023). However, many students face difficulties in understanding geometry concepts, mainly due to its abstract characteristics and high visualisation requirements (Sari et al., 2024; Septiani, 2024; Gal & Linchevski, 2010). Research shows that these challenges include difficulties in understanding formal symbols, problem solving, and application of concepts in real life (Santos-Trigo, 2024; Aprilia & Setiawan, 2021; Pratiwi & Anita, 2021). Therefore, an innovative, interactive, and context-based learning approach is needed to help students overcome these challenges, so that they can effectively develop their critical thinking, problem solving, and deductive reasoning skills in learning geometry.

Van Hiele's learning theory offers a comprehensive framework for understanding how students learn geometry (Usiskin, 1982). The theory consists of five levels of geometric understanding: visualisation, analysis, abstraction, formal deduction, and rigor. At the visualisation level, students recognise geometric shapes based on visual appearance without understanding their underlying properties. At the level of analysis, students begin to identify the properties and components of geometric shapes. The abstraction level involves understanding the relationships between these properties. At the formal deduction level, students begin to understand the role of deduction and axiomatic systems in geometry.


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Finally, at the rigour level, students are able to understand and work with higher abstractions in geometry, including axiomatic and non-Euclidean systems (Van Hiele, 1986). These levels help students understand properties and relationships in geometry gradually, starting from the recognition of visual shapes to understanding complex axiomatic systems.

The integration of ethnomathematics can support this theory by providing relevant cultural contexts, thus making learning more meaningful for students. For example, the exploration of geometric shapes in traditional architecture in traditional houses can be used to learn geometry with students' everyday experiences, while facilitating their understanding of the higher stages of geometric thinking (Simanjuntak et al., 2024; Yanti et al., 2024; Laukum et al., 2024). Ethnomathematics is a field of study that explores the relationship between mathematics and culture (D'Ambrosio, 1985). The term was first introduced by Ubiratan D'Ambrosio in the 1980s and encompasses the ways in which specific cultural groups understand, express and use mathematical concepts in their daily lives. By integrating ethnomathematics in learning, mathematics education can become more contextualised and meaningful to students (D'Ambrosio, 2006). Thus, the integration of ethnomathematics in education can be a bridge that connects formal mathematical concepts with students' cultural experiences, thus creating more inclusive, relevant and transformative learning.

Indonesia itself has cultural diversity that grows and develops in each region (Ansori & Iskandar, 2023). In the Special Region of Yogyakarta, there are many cultural artefacts that can be used as ethnomathematics studies, one of which is the Vredeburg Fort Museum. The Vredeburg Fort Museum in Yogyakarta is one of the historical sites rich in cultural and historical values (Santoso, 2021). The museum, which was originally built as a fort by the Dutch in the 18th century, now serves as a museum that documents the history of Indonesia's independence struggle. Using the Vredeburg Fort Museum as a learning context can provide a more in-depth and interesting experience for students in learning mathematical concepts, especially in geometry. An ethnomathematics-based learning approach involves using local cultural and historical contexts to teach mathematical concepts (Pratama & Yelken, 2024). By utilising the artefacts of Museum Benteng Vredeburg, students can learn about the concept of congruence in geometry in a more interesting and relevant way. Students can observe and analyse geometric shapes found in building structures and artefacts in the museum, and understand how these concepts were used in design and construction in the past.

Combining Van Hiele's learning theory with an ethnomathematics approach offers a comprehensive and contextualised approach to geometry learning. Van Hiele's theory provides a framework for understanding how students learn geometry (Pegg, 2014), while ethnomathematics provides a relevant and meaningful context for such learning (Harding, 2021). In this context, students are invited to recognise and analyse geometric shapes (Schenck & Nathan, 2024), especially in Museum Benteng Vredeburg, understand their properties, and relate them to the concept of congruence.

The use of Museum Benteng Vredeburg in geometry learning can provide valuable insights into how this approach can be applied. Students can be invited to conduct field explorations in the museum, observing geometric shapes found in buildings and artefacts, and discussing the geometric properties and relationships they find. In addition, students can also be given tasks that involve making geometric models based on their observations, as well as presenting their work to classmates. So a learning approach that integrates Van Hiele's theory and ethnomathematics has significant implications for mathematics education. By providing a context that is relevant and meaningful to students, this approach can increase students' engagement and motivation in learning mathematics (Machaba & Dhlamini, 2021). In addition, this approach can also help students develop a deeper and more comprehensive understanding of geometric concepts, as well as improve their reasoning ability (Ishikawa & Newcombe, 2021).

Various previous studies have examined the application of Van Hiele's theory in learning geometry with a focus on various approaches and learning contexts. Arnal-Bailera & Manero (2024) conducted a study that focused on defining and characterising level 5 of geometric reasoning according to Van Hiele's theory using Delphi methodology. The study highlighted that level 5 involves advanced abstract reasoning and the ability to understand and construct formal proofs in geometry. The findings suggest that achieving this level requires deep conceptual understanding as well as exposure to rigorous mathematical tasks and problems. This study provides insight into teaching strategies that can support students in achieving higher levels of geometric reasoning. Then research conducted by Watan & Sugiman (2018) examined the correlation between teachers' teaching methods and students' level of geometric thinking as described by Van Hiele's theory. It was found that effective teaching practices aligned with Van Hiele's levels significantly improved students' understanding and development of geometric thinking. This research emphasises the need for teachers to adapt their teaching strategies to students' cognitive developmental stages in geometry.

Another research conducted by Falupi, Gembong, and Handayani (2024) examined the level of geometry thinking of grade IX students on flat building materials using Van Hiele's theory in synchronous and asynchronous learning in terms of cognitive style. The results of this study indicate that the cognitive style of students affect the achievement of the level of geometric thinking, providing insight into the importance of the approach in accordance with the characteristics of students. Then Fauziah, Oktavia, and Alpian (2024) explored the application of Van Hiele's theory in learning quadrilateral geometry. This research highlights that Van Hiele's theory-based approach can help students understand geometry concepts gradually according to their level of thinking, emphasising the benefits of the hierarchical structure in this theory. Furthermore, Safira (2024) examined the effectiveness of Contextual Teaching and Learning (CTL) learning model with Van Hiele's theory approach in improving students' verbal reasoning ability in geometry. This research shows that context-based learning integrated with Van Hiele's theory is able to improve students' ability to understand geometry concepts in depth. And Pradana (2024) examined the effect of mathematics learning using Creative Visualisation techniques on students' geometry ability level. This study shows that the creative visualisation approach can help students reach higher levels of geometry thinking more effectively. These findings contribute to the understanding of how Van Hiele's theory can be applied to improve geometry education by aligning teaching practices with students' cognitive developmental stages.

Based on previous research, there is still no research that specifically integrates Van Hiele's theory in learning geometry with ethnomathematics-based, especially in the context of local culture such as Museum Benteng Vredeburg. Therefore, this study focuses on examining students' geometric thinking ability in learning ethnomathematics-based congruence with Van Hiele's theory approach, which is expected to make a new contribution to culture-based geometry learning. The purpose of this study is to determine the effectiveness of learning and how to increase the level of geometric thinking according to Van Hiele's theory. As well as how students' experiences of learning ethnomathematics-based kekongruenan at the Museum Benteng Vredeburg.

## Theotrical Perspective

### Van Hiele's Theory

Van Hiele's theory is a cognitive developmental model that explains how students learn and understand geometric concepts (Blair, 2004). The theory includes five levels of geometric understanding (Arnal-Bailera, & Manero, 2024): 1) Visualisation (Level 0) at this stage students recognise shapes based on their global appearance, not their geometric properties or attributes. 2) Analysis (Level 1), at this stage students begin to recognise the properties and geometric components of the shape. 3) Abstraction (Level 2), at this stage students understand the relationship between geometric properties and can form geometric definitions. 4) Formal Deduction (Level 3), at this stage students can develop logical arguments and understand the deductive structure of geometry. And finally, Rigor (Level 4), at this stage students understand the formalities and axiom system of geometry and are able to work with various geometric systems. Each of these levels builds a deeper and more complex understanding of geometry, allowing students to learn new concepts more effectively.

**Ethnomathematics**

D'Ambrosio (2001) defines ethnomathematics as mathematics practised among specific cultural groups, which includes mathematical concepts and techniques developed in various cultures to solve problems of everyday life. Ethnomathematics is a mathematical thinking, understanding and techniques integrated in the culture of a community (Tamur et al., 2022). The term "ethno" refers to an identifiable group of people and does not necessarily pertain to local communities or to linguistic identifications (Orduna-Malea et al., 2019). Contextualised mathematics learning can utilise culture as a relevant and engaging learning resource. The term that combines culture with mathematics is known as ethnomathematics (Darmayasa et al., 2018). Linking mathematics learning with culture certainly facilitates the process of learning mathematics itself, because students can more easily understand what topics are studied because they are related to daily cultural life (Sirate, 2012). Therefore, learning mathematics really needs an ethnomathematics approach to teach abstract mathematical concepts to students.

One way to make learning mathematics more meaningful is to relate it to the environment or culture embedded in students so that it becomes something real and meaningful in the minds of students (Polman, 2021). So that students do not quickly forget the mathematics material they learn, because these problems often appear in everyday life. In addition, students are accustomed to linking every mathematical material with the culture that applies in everyday life. According to Fajriyah (2018), ethnomathematics also supports mathematical literacy by facilitating students' ability to construct mathematical concepts as part of mathematical literacy based on students' knowledge of their sociocultural environment. Furthermore, Putri (2017) stated that the field of ethnomathematics includes mathematical ideas, thoughts and practices developed by all cultures. Ethnomathematics is used as a bridge between mathematics learning and culture that is able to provide knowledge and values that are better understood because they are related to culture in accordance with local traditions in mathematics learning (Ansori & Iskandar, 2023). This is because ethnomathematics offers local culture-based learning so that students can simultaneously recognise and explore the culture of their country.

According to Marsigit (2016) Ethnomathematics-based mathematics learning provides properties: (1) Providing and using various teaching aids, (2) learning mathematics in various places and opportunities. (3) Using mathematics for various purposes. (4) Developing an attitude of using mathematics as a tool to solve problems both at school and at home. (5) Appreciate the contribution of tradition, culture and art in the development of mathematics. (6) Helping students reflect on their mathematical activities.

***Museum Benteng Vredeburg* (Fort Vredeburg)**

Dutch colonialism in the archipelago affected almost all aspects of economic, social and cultural life. Indonesian culture that gradually "blended" with European (Dutch) culture gave rise to a new culture called Indis Culture (Soekiman 2011). In terms of architecture, the Dutch brought their influence to build European-style buildings and settlements Wihardyanto, D., & Sudaryono, S. (2020). The personality of the Javanese Nation (local genius) and the tropical climate in Indonesia also influenced the role of acculturation of these two cultures (Susanti, 2013). This is certainly an evolution that has developed in Indonesian culture.

The Indische Empire Style was introduced by H.W. Daendels who was assigned by the Dutch East Indies Government as governor-general in the 1808-1811 term (Handinoto Handinoto, 2008). As the name implies, Indische means "Indian". The name reflects a building style from Europe, especially the Netherlands. This style is a fusion of European culture with Indonesian culture and some from Chinese culture (Handinoto 2010). This style has certain characteristics that exist in each room in a building. One of the characteristics is a wide terrace with Greek columns at the end, such as Doric, Ionic, and Corinthian (Purnomo, Waani, and Wuisang 2017).

The Indische Empire Style can be found in one of the historical places, the Vredeburg Fort Museum Yogyakarta (Rizki, 2022). Moreover, this fort is the first generation to use the concept of related building styles. At the entrance gate area, the Greco-Roman style of the building is easily seen in the Doria-style pillars. After the entrance gate, there are two "flanking" buildings that are also Greek Renaissance in style. We can also see other Dutch-style features such as large windows and thick walls. In addition, in certain buildings there is a form of roof top decoration (Nok Acroterie) like a short carved pseudo chimney (Soekiman 2011). The roof shapes are mostly Javanese, such as joglo and limasan styles.

## Materials And Methods

### Study area

This research uses quantitative and qualitative approaches. The quantitative approach was used to determine the effect of ethnomathematics learning on geometry thinking skills and describe the increase in geometry thinking levels based on Van Hiele's theory after learning conicongruence ethnomathematics-based at Museum Benteng Vredeburg. A qualitative approach was used to describe students' experiences of classroom learning. The research subjects were class VIII students in one of the junior high schools in Yogyakarta, who were purposively selected to ensure relevance to the research topic.

### Procedures

The research instruments used include participatory observation, in-depth interviews, and pretest-posttest tests of geometry thinking skills. Observations were made to observe student interactions during the learning process, while in-depth interviews aimed to gain a deeper understanding of students' geometric thinking. The pretest-posttest test was used to measure students' level of understanding of the concept of congruence according to the stages of Van Hiele's theory. The research procedure starts from the preparation stage, which is the identification of artefacts of Museum Benteng Vredeburg on the subject of congruence, as well as the preparation of ethnomathematics-based lesson plans. Furthermore, in the implementation stage, observations were made during learning in the museum, interviews with students after learning activities, and tests of geometric thinking skills.

### Data analysis

The data analysis stage involved collecting data from observations, interviews, and tests, which were then analysed descriptively to identify students' geometry thinking stages based on Van Hiele's theory.

## Results And Discussion

### Ethnomathematics of Museum Benteng Vredeburg on the Matter of Conicruence

This research focuses on the Ethnomathematics of the fort at Fort Vredeburg. This fort is block-shaped and each corner of the fort is a pentagonal prism. When viewed from above, the basic shape of Fort Vredeburg is a square which then in each corner there is another congruent regular pentagon. The following is a picture of the Vredeburg Fort Museum looking from the front and looking from above, as well as its sketch and ethnomathematics review of congruence material.

**Figure 1.** Fort Vredeburg from Above.

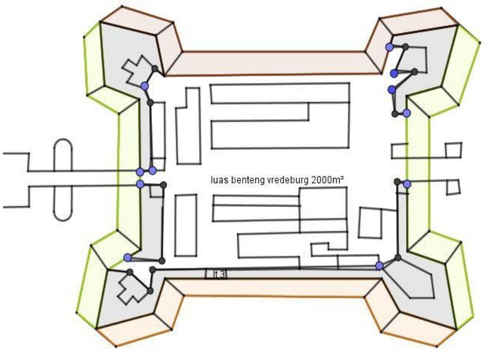



**Figure 2.** Sketch of Fort Vredeburg from Above

The sketch of Fort Vredeburg features congruences that can be observed in some of its architectural elements. One example is the pentagon shape found on the four bastions at the corners of the fort. Each of these bastions has an identical size and shape, with the same angle and side length, so it can be said that the bastions are congruent shapes. In addition, congruence is also seen in the structure of the buildings inside the fort. The buildings show congruence in their basic design, reflecting a uniform comparison of proportions. This reinforces the sense of harmony and order in the overall architectural design of the fort.

**Figure 3.** Fort Vredeburg from the Front

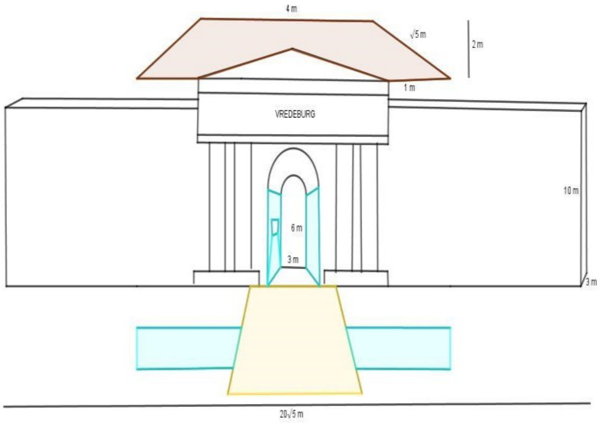
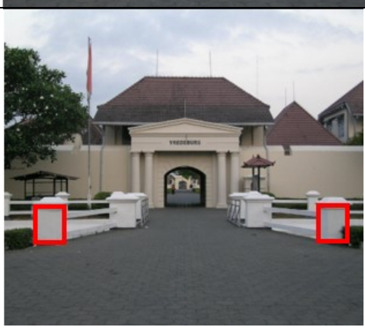



**Figure 4.** Sketch of Fort Vredeburg from the Front

The picture above shows congruence in some of the design elements. One of them is the doorposts at the front. These columns are symmetrically arranged and have identical shapes and sizes, so they can be said to be congruent. This uniformity in size and shape creates visual harmony at the front door. In addition, congruence is also seen in the fence supports in the front area. The small triangular shapes of these supports, both at the front of the building and on the ramps, show congruence of angles and similarity of side lengths. This makes the railing supports a consistent example of congruity, adding a regular aesthetic to the overall building design.

**Table 1.** Ethnomathematics of Conicruence at Benteng Vredeburg Museum.

| Image | Maths Concepts |
|---|---|
|  | There are 4 pentagon shapes in each corner of the museum. The corresponding sides and corresponding angles are the same size so the 4 pentagons are congruent. Two or more charts are called congruent if all their corresponding sides and all their corresponding angles are of the same size. |
|  | There are 2 rectangular shapes viewed from the top of the Museum. The corresponding sides and corresponding angles are the same size so the 2 rectangles are congruent. Two or more charts are called congruent if all their corresponding sides and all their corresponding angles are of the same size. |
|  | There are 2 triangular shapes on the roof structure of the building seen from the top of the Museum. The corresponding sides and corresponding angles are the same size so the 2 rectangles are congruent. Two or more charts are called congruent if all their corresponding sides and all their corresponding angles are of the same size. |
|  | There are 4 rectangular shapes in each corner of the museum. The corresponding sides and corresponding angles are the same size so the 4 rectangles are congruent. Two or more charts are called congruent if all their corresponding sides and all their corresponding angles are of the same size. |
|  | There are 2 rectangular shapes on the fence. The corresponding sides and corresponding angles are the same size so the 2 rectangles are congruent. Two or more charts are called congruent if all their corresponding sides and all their corresponding angles are of the same size. |

**Effectiveness of Ethnomathematics Learning**

The assessment of geometric thinking ability is based on the pretest and posttest scores of ethnomathematics-based congruence learning. The descriptive analysis results show that the pretest results (mean = 54.32, standard deviation = 16.51) are compared to the posttest results (mean = 74.66, standard deviation = 17.66). From this result shows that the average score of geometry thinking ability after ethnomathematics-based learning is higher than before learning. And the results of inferential analysis show that there is a significant difference between before and after applying ethnomathematics-based congruence learning (F-value = 21.30, p-value = 0.0000234). The significance value obtained is 0.0000234, which is $p < 0.05$, so H0 is rejected. Thus, ethnomathematics-based congruence learning is effectively applied in terms of geometric thinking ability.

**Thinking Ability**

The following table presents the pretest and posttest results of ethnomathematics-based congruence learning based on the level of geometric reasoning according to Van Hiele's theory.

**Table 2.** Pretest and Posttest Results Based on Geometric Thinking Levels.

| Pretest | | Posttest | |
|---|---|---|---|
| Thinking level | Total | Thinking level | Total |
| Level 0 | 0 | Level 0 | 0 |
| Level 1 | 9 | Level 1 | 2 |
| Level 2 | 11 | Level 2 | 3 |
| Level 3 | 7 | Level 3 | 12 |
| Level 4 | 2 | Level 4 | 12 |
| Total | 29 | Total | 29 |

The research results presented in the table above show a significant increase in students' geometry thinking skills after participating in ethnomathematics-based learning. The pretest results showed that most students were at the low to medium level of reasoning, with details of 9 students at Level 1 (analysis) and 11 students at Level 2 (abstraction). Only 7 students were able to reach Level 3 (formal deduction), while there were only 2 students at Level 4 (rigour). There were no students at Level 0, either before or after learning, which indicates that all students have a basic level of geometry thinking.

After the lesson, the posttest data showed a very significant change. The number of students in Level 1 dropped dramatically from 9 to only 2, indicating that the majority of students who were previously in this level managed to improve their ability to a higher level. Similarly, the number of students in Level 2 also reduced significantly from 11 to only 3 students, indicating their move to Level 3 or even Level 4.

At Level 3 (formal deduction), the number of students increased from 7 to 12, indicating an increase in students' geometric reasoning ability after learning. The most striking increase occurred at Level 4 (rigour), where the number of students increased from only 2 to 12. This shows that ethnomathematics-based learning has a very positive impact on students with higher geometric reasoning ability.

Ethnomathematics-based learning proved effective in improving students' geometric reasoning skills. There was a significant shift from low (level 1) and medium (level 2) to higher levels of thinking (level 3 and level 4). This result shows that the learning approach that integrates cultural values is able to strengthen students' understanding, while increasing their thinking ability to a more complex level. This research provides strong evidence that ethnomathematics-based learning can be one of the innovative solutions to improve the quality of mathematics education, especially in certain cultural contexts.

**Improved Geometric Thinking Level**

The following table presents the results of increasing the level of geometric thinking according to Van Hiele's theory in learning ethnomathematics-based congruence at Museum Benteng Vredeburg.

**Table 3.** Improvement of Geometric Thinking level.

| | | Posttest | | | | |
|---|---|---|---|---|---|---|
| | | Level 0 | Level 1 | Level 2 | Level 3 | Level 4 |
| Pretest | Level 0 | 0 | 0 | 0 | 0 | 0 |
| | Level 1 | 0 | 2 | 1 | 2 | 4 |
| | Level 2 | 0 | 0 | 2 | 4 | 5 |
| | Level 3 | 0 | 0 | 0 | 6 | 1 |
| | Level 4 | 0 | 0 | 0 | 0 | 2 |

The table above presents data on pretest and posttest results related to ethnomathematics-based learning of congruence, with reference to pretest results before learning then the level of geometric thinking has increased level after ethnomamatics-based learning.

At the beginning of learning (pretest), students showed a varied distribution of geometric reasoning ability, ranging from Level 1 (Analysis) to Level 4 (Rigour). No students were found at Level 0 (Visualisation). After learning (posttest), there was a significant increase in students' geometric reasoning ability, which can be seen from the movement of students to higher levels.

Students who started at Level 1 showed very positive progress. Out of a total of 9 students, 2 students remained at Level 1, while the other 7 students managed to advance to higher levels. A total of 1 student rose to Level 2 (Informal Deduction), 2 students rose to Level 3 (Deduction), and 4 students reached Level 4 (Rigor), which is the highest level in geometric reasoning. This shows that ethnomathematics-based learning is able to provide effective stimulation to improve students' geometric thinking skills. Students who started at Level 2 also showed significant results. Out of 11 students, 2 students remained at Level 2, but most improved. A total of 4 students moved up to Level 3, and 5 students reached Level 4. This indicates that this learning is effective in encouraging students to develop their deductive abilities and understand geometry concepts more deeply. Students who started at Level 3 were already at a fairly high level of ability. Out of 7 students, 6 students remained at Level 3, and 1 student made it to Level 4. The stability at Level 3 shows that ethnomathematics-based learning is also able to maintain students' already high ability, while providing additional encouragement for those who are able to go further. Students who started at Level 4 (2 in total) remained at this level after the learning. This shows that the learning was successful in maintaining students' competence at the highest level, although there was no further improvement as they were already at the maximum level.

Ethnomathematics-based congruence learning shows high effectiveness in improving students' geometric reasoning ability. Students who started from low levels (Level 1 or Level 2) showed significant improvement towards higher levels (Level 3 or Level 4). Meanwhile, students who are already at a high level are able to maintain their abilities well. This learning not only promotes conceptual understanding, but also strengthens deductive thinking and rigour in geometry. The findings suggest that ethnomathematics-based learning, which links mathematical concepts with students' cultural context, can be an effective method to improve geometric reasoning according to Van Hiele's theory.

**Student experience**

Learning ethnomathematics-based congruence with the context of Museum Benteng Vredeburg provides a unique and in-depth learning experience for students. Through this approach, students not only learn the mathematical concept of congruence formally, but also understand its connection with local cultural heritage. The results of this study showed some important experiences felt by students during the learning process, which then contributed to the improvement of geometric reasoning levels based on Van Hiele's theory.

At the beginning of the lesson, students were introduced to various geometric shapes found in

Museum Benteng Vredeburg, such as triangles, quadrilaterals and triangles in the cross-sectional images of the museum from above and from the front. This activity utilises students' visual observations to build early experiences related to geometry. Students feel interested because they can relate learning to something real and familiar in everyday life.

For students who were previously at Level 1 (Analyse), this experience helps them understand the relationships between geometric shapes through deep observation. They begin to identify certain elements, such as sides, angles and symmetry, that they had not realised before. During the learning process, students are invited to discuss in groups to analyse patterns of congruence found in various museum artefacts and structures. In this activity, students at Level 2 (Informal Deduction) begin to develop the ability to make connections between geometric elements and apply the principle of congruence. This experience helps students to formulate simple mathematical arguments, such as determining the similarity of shapes in two different structures based on angle sizes and side lengths. Some students claimed that discussing with peers helped them to see patterns that they had not previously understood.

For students at Level 3 (Deduction), the ethnomathematics-based approach provides space to develop deductive understanding. Learning activities involved students in constructing simple mathematical proofs related to congruence, for example by using direct measurements on pictures of museum artefacts provided by the teacher. Students at this level stated that the experience encouraged them to think logically and systematically, especially when they were asked to explain the reasons behind their conclusions. This strengthened their understanding of congruence. Students who are already at Level 4 (Rigor) are able to connect the geometry concept of congruence with the cultural context in an abstract way. They not only understood the concept deeply, but were also able to explain the connection with the cultural patterns in Museum Benteng Vredeburg. This experience made students feel more confident in applying mathematical concepts to complex situations. They mentioned that this learning provided new insights into how culture can be used to learn mathematics.

Students stated that this ethnomathematics-based learning made them feel more motivated to learn. The cultural context used provides a unique appeal, so students feel that mathematics is not just about numbers and formulas, but can be visualised in a real context. Students stated that they felt more comfortable learning in an environment that allowed visual exploration and social interaction, compared to traditional learning.

Ethnomathematics-based congruence learning using the context of Museum Benteng Vredeburg succeeded in creating a rich and meaningful learning experience for students. This approach not only improves students' geometric reasoning level according to Van Hiele's theory, but also strengthens students' engagement in the learning process. By linking mathematics with local culture, students gain greater insight into the relevance of mathematics in everyday life, as well as feel more enthusiastic and motivated in learning geometry, especially the subject of congruence.

## Discussion

Vredeburg Fort Museum's ethnomathematics-based congruency learning is effective in improving students' geometric thinking skills based on Van Hiele's theory. This approach connects mathematical concepts with local culture, such as patterns and geometric shapes found in the museum, which helps students build concrete mathematical understanding before stepping into abstract understanding. This shows that ethnomathematics-based learning not only enriches students' learning experience, but also encourages the mastery of geometric concepts gradually according to the stages of thinking outlined in Van Hiele's theory.

Previous research supports the results of this study. The development of geometry thinking skills requires structured learning through five stages: information, directed orientation, explanation, free orientation, and integration. These stages help students progress from low to higher levels (Naufal et al, 2021; Perkasa, 2024; Tarlina & Madawistama, 2024). In addition, the ethnomathematics approach can be a bridge between mathematics learning and local culture, making it easier for students to understand concepts through real contexts. Ethnomathematics helps integrate mathematics with everyday experiences, such as design, measurement, and cultural patterns, which are relevant in learning activities (Kurniawan, 2024; Lestari et al., 2024; Nursanti et al., 2024). Cultural contexts make students feel more connected to the subject matter thus increasing their engagement in the learning process. Students feel that the material presented is more relevant to their lives, so they are more enthusiastic about learning mathematics.

The progress of students' geometric thinking from Level 1 to Level 4 illustrates that ethnomathematics-based learning provides a significant stimulus for students' cognitive development. Many students who started from low levels showed rapid progress to reach Level 3 (Deduction) and even Level 4 (Rigour). This proves that this approach not only strengthens the basis of mathematical understanding but also encourages students to think deductively and rigorously. Learning that involves group discussion and collaboration also supports students' learning experience. Through discussion, students can share ideas and viewpoints with classmates, which helps them enrich their understanding and strengthen critical thinking skills. This collaboration allows lower-level students to learn from higher-level peers, creating an inclusive learning environment. The study also found that students who were already at Level 4 (Rigour) tended to maintain their proficiency without showing further improvement. This indicates that ethnomathematics-based learning is effective in helping early to mid-level students to progress to higher levels.

## Conclusions

Learning about congruence based on ethnomathematics of Museum Benteng Vredeburg proved to be effective in helping students understand the concept of congruence contextually. This approach accelerates students' transition to higher levels of geometric reasoning according to Van Hiele's theory. Many students who were initially at Level 1 and Level 2 successfully reached Level 3 (Deduction) and Level 4 (Rigour), showing significant improvement in their cognitive abilities. In addition, the use of local cultural context had a positive impact on students' motivation and engagement in the learning process.

While this approach was effective, there is still room for further research, particularly in developing additional strategies to help students at Level 4 maximise their potential. Future research could explore other innovative approaches that can expand geometry insights and skills in students, particularly those already at the highest levels. Thus, ethnomathematics-based learning is not only a means to improve mathematical ability, but also an important tool to preserve and appreciate local culture through education

**Conflict of Interest:** The authors declare that there are no conflicts of interest concerning the publication of this article.

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