

Batik Pattern Representation of SITR Mathematical Model on Tuberculosis Transmission in Aceh Province

Gicela Viga Gita Wisesa*, Jamila Maulida Sholichati, Riska Fajry Rahmadhanie, Sugiyanto

Mathematics Department, Faculty of Science and Technology, UIN Sunan Kalijaga,
Jl. Marsda Adisucipto No 1 Yogyakarta 55281, Indonesia. Tel. +62-274-540971, Fax. +62-274-519739.

Corresponding author*

gicela.ss39@gmail.com

Abstract: *Mycobacterium tuberculosis* is the causative agent of tuberculosis (TB), which continues to rank among the most dangerous infectious diseases in Aceh Province. In order to characterize the dynamics of TB transmission, treatment, and recovery, this study intends to develop and evaluate the stability of the SITR (Susceptible–Infectious–Treatment–Recovery) mathematical model. Four compartments are created by the model: susceptible, infectious, treatment, and recovery. The Aceh Provincial Health Office provided secondary data. The basic reproduction number (R_0) and the disease-free and endemic equilibrium points were determined in order to conduct the stability analysis. The findings demonstrate that the SITR model has two equilibrium points and maintains stability when there is no sickness, with $R_0 = 0.124$, suggesting that there isn't a population breakout. Additionally, to graphically depict the dynamics of TB transmission and recovery in Aceh, the model simulation was converted into a batik motif design. As a result of the disease's controlled spread, the number of susceptible people gradually declines over time, while the number of people receiving treatment and recovering rises. These results show that the SITR model has possibilities for imaginative visualization using traditional art forms like batik in addition to offering a trustworthy analytical foundation for illness modeling.

Keywords: Mathematical Modeling, SITR Model, Stability Analysis, Tuberculosis.

Abbreviations: TB – Tuberculosis; SITR – Susceptible-Infective-Treatment-Recovery; DFE – Disease-Free Equilibrium; EE – Endemic Equilibrium; R_0 – Basic Reproduction Number.

Introduction

Tuberculosis (TB) is a chronic infectious disease caused by the bacterium *Mycobacterium tuberculosis*. The disease mainly attacks the lungs but can also spread to other organs such as the bones, kidneys, and brain. Until now, TB remains a major global health problem. According to the Indonesian Ministry of Health, Indonesia ranks second in the world after India for the highest number of TB cases in 2024. The WHO Global TB Report 2024 estimates that Indonesia has around one million new TB cases each year, with a treatment success rate of approximately 85%.

TB transmission occurs through droplets released by infected individuals when coughing or sneezing and can easily spread to people with weakened immune systems. Contributing factors

include high population density, poor socioeconomic conditions, and limited access to healthcare services. The growing number of multidrug-resistant tuberculosis (MDR-TB) cases further complicates disease control since it requires longer treatment durations and more intensive supervision.

Efforts to control TB require not only medical treatment but also analytical and mathematical approaches to understand the dynamics of its transmission. Epidemiological mathematical modeling has become an essential tool in describing, predicting, and managing the spread of infectious diseases. Classical models such as the Susceptible–Infectious–Recovered (SIR) model have been widely applied, but for TB, the model has evolved into the Susceptible–Infectious–

Treatment–Recovered (SITR) model to better represent the treatment stage that is unique to this disease.

Aceh Province is one of the regions in Indonesia with a relatively high number of TB cases. Data from the Aceh Provincial Health Office (2023) report that the number of cases exceeds 8,000 per year. This situation highlights the urgency of predictive efforts using mathematical models that can assist in regional health policy planning. The SITR model offers a framework to represent population interactions between susceptible, infected, treatment, and recovery groups, providing insights into how interventions may affect TB transmission rates. Previous research by Zahwa et al. (2022) applied the SITR model to analyze TB spread in Aceh and found that the system reached a disease-free equilibrium with $R_0 < 1$, suggesting that the infection could be controlled through consistent treatment.

Several other Indonesian studies have also explored TB modeling to reflect local conditions and health strategies. Simorangkir et al. (2021) developed a mathematical model that includes observed treatment and vaccination effects, while Roudhotillah and Chandra (2022) focused on analyzing model stability using eigenvalue and reproduction number methods. Meanwhile, Amatillah et al. (2022) incorporated vaccination and latency factors to evaluate how interventions influence TB control outcomes. These studies emphasize that mathematical modeling is not only theoretical but also directly applicable for understanding disease control in Indonesia.

In addition to mathematical analysis, this study introduces a creative approach that integrates science with Indonesian cultural heritage. The SITR model simulation is represented through batik pattern visualization, symbolizing the dynamic process of infection, treatment, and recovery. Batik patterns, known for their repetitive and harmonious forms, can be interpreted as visual

metaphors for the cyclical behavior observed in epidemiological models. This approach aims to communicate scientific findings in a form that is both informative and culturally meaningful for the Acehnese community.

Therefore, this research aims to (1) formulate and analyze the SITR mathematical model of tuberculosis transmission in Aceh Province, (2) determine the equilibrium points and basic reproduction number (R_0) to assess the system's stability, and (3) visualize the simulation results as batik-inspired patterns representing the transition between infection, treatment, and recovery. By combining mathematical analysis with creative visualization, this study seeks to strengthen both scientific understanding and local cultural appreciation in representing public health issues in Aceh.

Materials And Methods

Study area

This research focuses on Aceh Province, Indonesia. Aceh was chosen because the TB case rate in the province showed a significant fluctuating trend between 2018 and 2021. According to data from the Central Statistics Agency (BPS) and the Aceh Health Office, in 2018, the number of TB cases reached 8,471 cases. In 2019, this number increased to 8,647 cases, and then decreased to 8,372 cases in 2020.

This variation in cases indicates that the spread of TB in Aceh has not been fully controlled, despite interventions through long-term treatment and public health programs. Furthermore, population density, social mobility, and access to health services can also influence TB transmission patterns. Therefore, mathematical modeling is used to understand the dynamics of TB transmission in the Aceh region.

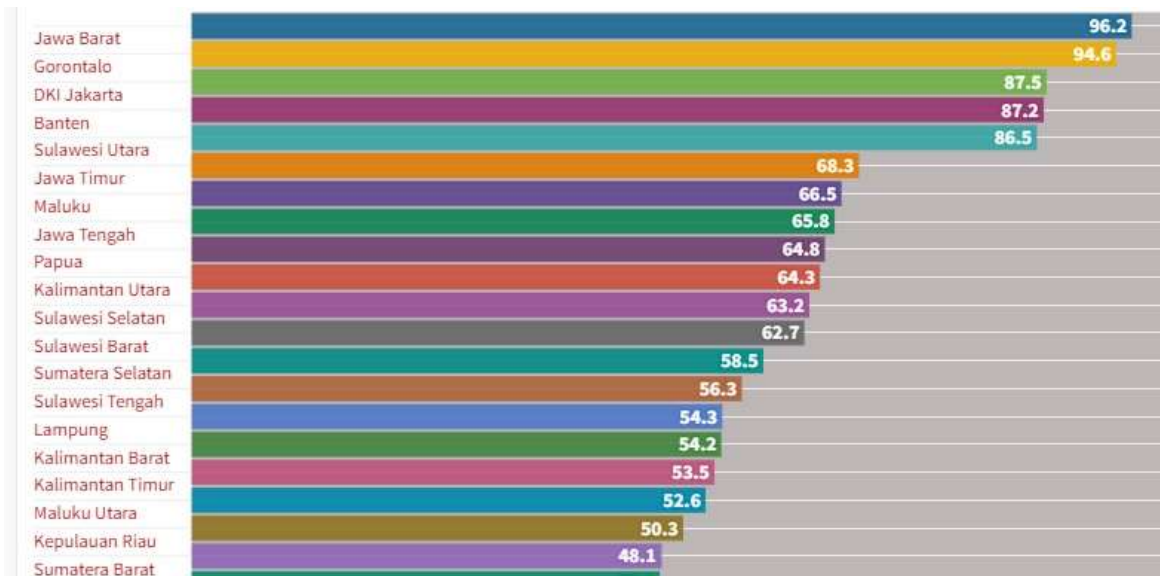


Figure 1. The rate of spread of tuberculosis in Indonesia

Procedures

This research uses a theoretical study method based on a mathematical model with a determinant analysis approach. The method aims to build a model that can represent interactions between individuals in a population potentially infected with tuberculosis and to analyze the stability of its transmission system.

Sub-procedures-1: Formula of SITR Model

The epidemiological model used is SITR (Susceptible–Infective–Treatment–Recovery), a development of the SIR model. This model describes the movement of individuals in a population from susceptible (S), infected (I), treatment (T), and recovered (R). The four population compartments in this model are described as follows:

1. $S(t)$: the number of susceptible and uninfected individuals, but potentially infectious.
2. $I(t)$: the number of infected individuals who can transmit the disease.
3. $T(t)$: the number of individuals undergoing treatment.
4. $R(t)$: the number of individuals who have recovered.

The system of differential equations used to model the dynamics of disease spread is expressed as follows:

$$\frac{dS}{dt} = \theta - \beta IS - \mu S$$

$$\frac{dI}{dt} = \beta IS - \alpha I - \mu I$$

$$\frac{dT}{dt} = \alpha I - \gamma T - \mu T$$

$$\frac{dR}{dt} = \gamma T - \mu R$$

with parameter descriptions:

- θ : population birth rate per unit time,
- β : rate of infection transmission between individuals,
- μ : natural death rate of the population,
- α : rate of individuals moving from the infected group to the treatment group,
- γ : rate of recovery of individuals from the treatment group to the recovered group.

The basic assumptions of the model include:

- a. The population $N(t) = S(t) + I(t) + T(t) + R(t)$ is assumed to be constant.
- b. Individuals born directly into the susceptible group.
- c. There is no significant immigration or emigration of the population.
- d. Recovered individuals have permanent immunity.

- e. The infection rate is proportional to the product of the number of susceptible and infected individuals.
- f. The treatment rate is assumed to be constant throughout the observation period.

Sub-procedures-2: Analysis of Eigenvalues and Equilibrium

To determine system stability, analysis is performed by determining equilibrium points and analyzing their eigenvalues. The model has two main types of equilibrium points:

1. A disease-free equilibrium point, where no individuals are infected.
2. An endemic equilibrium point, where the disease remains stable in the population.

Equilibrium points are determined by setting the derivatives of the system of equations equal to zero and finding the constant values of S, I, T, and R. The steps are as follows:

$$\begin{cases} 0 = \theta - \beta IS - \mu S \\ 0 = \beta IS - \alpha I - \mu I \\ 0 = \alpha I - \gamma T - \mu T \\ 0 = \gamma T - \mu R \end{cases}$$

- From equation 4: $0 = \gamma T - \mu R \rightarrow R = \frac{\gamma}{\mu} T$
- From equation 3: $0 = \alpha I - (\gamma + \mu) T \rightarrow T = \frac{\alpha}{\gamma + \mu} I$

So, T and R can be explain from I:

$$T = \frac{\alpha}{\gamma + \mu} I, R = \frac{\gamma}{\mu} T = \frac{\gamma \alpha}{\mu(\gamma + \mu)} I$$

- From equation 2: $0 = I(\beta S - \alpha - \mu)$
 $\rightarrow$ Solution: $I = 0$ or $\beta S - (\alpha + \mu) = 0$

Two case:

- i. *Disease-free equilibrium (DFE):* $I = 0$.

Substitution $I = 0$ to equation 1:

$$0 = \theta - \beta \cdot 0 \cdot S - \mu S \rightarrow \theta - \mu S = 0 \rightarrow S = \frac{\theta}{\mu}$$

Since $I = 0$, then $T = 0, R = 0$

So,

$$DFE = (S^*, I^*, T^*, R^*) = \left(\frac{\theta}{\mu}, 0, 0, 0\right)$$

- ii. *Endemic equilibrium (nontrivial):* jika $I \neq 0$ then it happens:

$$\beta S - (\alpha + \mu) = 0 \rightarrow S^{**} = \frac{\alpha + \mu}{\beta}$$

From the equation 1 in Equilibrium:

$$0 = \theta - \beta IS = \mu S$$

Substitution $S = S^{**} = \frac{\alpha + \mu}{\beta}$:

$$0 = \theta - \beta I^{**} \cdot \frac{\alpha + \mu}{\beta} - \mu \frac{\alpha + \mu}{\beta}$$

$$= \theta - (\alpha + \mu) I^{**} - \mu \frac{\alpha + \mu}{\beta}$$

$$(\alpha + \mu) I^{**} = \theta - \mu \frac{\alpha + \mu}{\beta}$$

$$I^{**} = \frac{\theta - \mu \frac{\alpha + \mu}{\beta}}{\alpha + \mu} = \frac{\beta \theta - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}$$

$$\text{Next, } T^{**} = \frac{\alpha}{\gamma + \mu} I^{**}, R^{**} = \frac{\gamma \alpha}{\mu(\gamma + \mu)} I^{**}$$

So,

$$\begin{aligned} \text{Endemik: } S^{**} &= \frac{\alpha + \mu}{\beta}, I^{**} = \frac{\beta \theta - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}, T^{**} \\ &= \frac{\alpha}{\gamma + \mu} I^{**}, R^{**} = \frac{\gamma \alpha}{\mu(\gamma + \mu)} I^{**} \end{aligned}$$

A. Conditions of Existence of Endemic Equilibrium

From the numerator:

$$I^{**} > 0 \leftrightarrow \beta \theta - \mu(\alpha + \mu) > 0$$

Define the reproduction number R_0 :

$$S_{DFE} = \frac{\theta}{\mu}, R_0 = \frac{\beta S_{DFE}}{\alpha + \mu} = \frac{\beta(\frac{\theta}{\mu})}{\alpha + \mu} = \frac{\beta \theta}{\mu(\alpha + \mu)}$$

Look at the equation:

$$\beta \theta - \mu(\alpha + \mu) > 0 \leftrightarrow \frac{\beta}{\mu(\alpha + \mu)} > 1 \leftrightarrow R_0 > 1$$

So,

The endemic ($I^{**} > 0$) $\leftrightarrow R_0 > 1$

If $R_0 < 1$ then $I^{**} \leq 0$ so only DFE is relevant

Next, the system is linearized around the equilibrium point by constructing a Jacobian matrix. The eigenvalues of this matrix are used to determine the stability of the equilibrium point. If all eigenvalues are negative, the system is said to be asymptotically stable. However, if there are one or more eigenvalues with a positive real part, the point is unstable.

Sub-procedures-3: The Basic Reproduction Number (R_0) by a single infected individual in a fully susceptible population. The R_0 value is calculated from the constant-part Jacobian matrix at the disease-free equilibrium point. The following is the formulation of the Jacobian matrix for this case:

1) General Jacobian Formula

Variable order (S, I, T, R). Calculate the partial differential for every variable:

$$J(S, I, T, R) = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial I} & 0 & 0 \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial I} & 0 & 0 \\ 0 & \alpha & -(\gamma + \mu) & 0 \\ 0 & 0 & \gamma & -\mu \end{pmatrix}$$

when $f_1 = \theta - \beta IS - \mu S$, $f_2 = \beta IS - \alpha I - \mu I$.

Calculate the differential:

$$\frac{\partial f_1}{\partial S} = -\beta I - \mu, \quad \frac{\partial f_1}{\partial I} = -\beta S,$$

$$\frac{\partial f_2}{\partial S} = -\beta I, \quad \frac{\partial f_2}{\partial I} = \beta S - \alpha - \mu$$

So

$$J = \begin{pmatrix} -\beta I - \mu & -\beta S & 0 & 0 \\ \beta I & \beta S - (\alpha + \mu) & 0 & 0 \\ 0 & \alpha & -(\gamma + \mu) & 0 \\ 0 & 0 & \gamma & -\mu \end{pmatrix}$$

2) Stability of DFE

Substitution of DFE: $S = \frac{\theta}{\mu}$, $I = 0$, $T = 0$, $R =$

0

Plug to Jacobian formula:

$$J_{(DFE)} = \begin{bmatrix} -\mu & -\beta \frac{\theta}{\mu} & 0 & 0 \\ 0 & \beta \frac{\theta}{\mu} - (\alpha + \mu) & 0 & 0 \\ 0 & \alpha & -(\gamma + \mu) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$$

Due to the triangular shape of the block, the eigenvalues are easily obtained as diagonal or block-eigenvalues:

From the row/column 1: $\lambda_1 = -\mu$

From the row/column 2: $\lambda_2 = \beta \frac{\theta}{\mu} - (\alpha + \mu)$

Write λ_2 into R_0 :

$$\lambda_2 = (\alpha + \mu) \left(\frac{\beta \theta}{\mu(\alpha + \mu)} - 1 \right) = (\alpha + \mu)(R_0 - 1)$$

From sub-block 3 until 4 eigenvalues:

$\lambda_3 = -(\gamma + \mu)$ dan $\lambda_4 = -\mu$

So, λ_2 explain the stability of DFE:

If $R_0 < 1$ then $\lambda_2 < 0$ and the result of the eigenvalues is negative (DFE is stable)

If $R_0 > 1$ then $\lambda_2 > 0$ and the result of the eigenvalue positive (DFE can't stable)

3) Endemic Stability Equilibrium

Procedures:

Using Jacobian Formula $J(S, I, T, R)$.

Substitution $S^{**}, I^{**}, T^{**}, R^{**}$ to J .

Remind:

$$S^{**} = \frac{(\alpha + \mu)}{\beta}, I^{**} = \frac{\beta \theta - \mu(\alpha + \mu)}{\beta(\alpha + \mu)}, T^{**} = \frac{\alpha}{\gamma + \mu} I^{**}$$

After substitution, found the matrix of 4×4 . The rest comes from the top of 2×2 blocks.

$$A = \begin{bmatrix} -\beta I^{**} - \mu & -\beta S^{**} \\ \beta I^{**} & \beta S^{**} - (\alpha + \mu) \end{bmatrix}$$

Noted: in endemic using $\beta S^{**} - (\alpha + \mu) = 0$ (because this using equilibrium condition in case $I \neq 0$, so the element became = 0. So, the 2×2 blocks:

$$A = \begin{bmatrix} -\beta I^{**} - \mu & -(\alpha + \mu) \\ \beta I^{**} & 0 \end{bmatrix}$$

Calculate the characteristics of polinomial $\det(\lambda I - A) = 0$

$$A = \begin{bmatrix} \lambda + \beta I^{**} + \mu & \alpha + \mu \\ -\beta I^{**} & \lambda \end{bmatrix} = 0$$

Calculate the determinant:

$$\lambda(\lambda + \beta I^{**} + \mu) + (\alpha + \mu)\beta I^{**} = 0$$

$$\lambda^2 + (\beta I^{**} + \mu)\lambda + (\alpha + \mu)\beta I^{**} = 0$$

Using rules of Routh-Hurwitz for polinomial $\lambda^2 + \alpha_1\lambda + \alpha_0$:

Condition that all roots have a negative real part $\alpha_1 > 0$ and $\alpha_0 > 0$.

There is:

$$\alpha_1 = \beta I^{**} + \mu, \quad \alpha_0 = (\alpha + \mu)\beta I^{**}$$

Since $\beta > 0, \mu > 0, \alpha + \mu > 0$ and for the endemic we have $I^{**} > 0$, then $\alpha_1 > 0$ and $\alpha_0 > 0$ proved. So both square roots have negative real parts.

Since sub blocks 3 to 4 give negative roots, the whole system has eigenvalues with negative real part meaning the local endemic equilibrium is asymptotically stable (under biological conditions the parameters are positive and $I^{**} > 0$ i.e $R_0 > 1$).

Results And Discussion

Results and Discussion should be written as a series of connecting sentences, however, for manuscript with long discussion should be divided into subtitles. Results should be clear and concise.

Simulation and Interpretations

Numerical simulations were conducted to analyze the dynamics of tuberculosis (TB) spread in Aceh Province using the SITR (Susceptible-Infective-

Treatment-Recovery) model. The solution of the system of differential equations was performed using MATLAB R2007b software, as it has advanced numerical computational capabilities and built-in functions that support the integration of nonlinear differential systems such as ode45.

Before start the simulation, write the initial condition on the table.

Table 1. Initial Condition

Variable	Absolut	Proportion
$S(0)$	5280000	0.990
$I(0)$	26665	0.005
$T(0)$	13332	0.0025
$R(0)$	13332	0.0025

This simulation used parameters tailored to reflect the conditions of TB spread in Aceh, namely:

Table 2. Parameters

Parameter	Description	Score
θ	Birth Rate	0.012
β	Infection Rate	0.18
μ	Natural Death Rate	0.013
α	Treatment Rate	0.002
γ	Cure Rate	1.5

The simulation steps are carried out by compiling the system's differential function in a MATLAB script and calling it using the ode45 solver, which is a Runge-Kutta method of order 4(5) which is suitable for continuous nonlinear systems such as epidemiological models. The output from MATLAB is a graph of the relationship between time and the population proportion of each compartment, as shown in Figure 1.

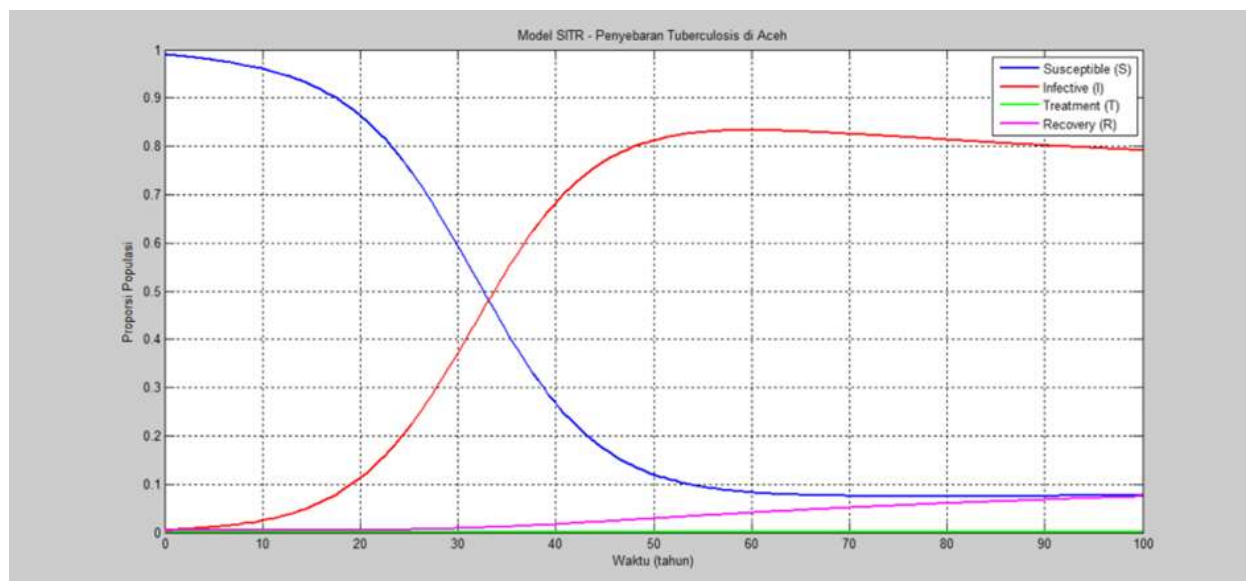


Figure 2. Result of SITR

The simulation results show the population dynamics in the four SITR compartments as follows:

1. Susceptible Compartment (S)

The blue curve in Figure 1 shows a sharp decline in the susceptible population from the beginning until around year 40. This curve indicates that a large proportion of initially healthy individuals become infected because the transmission rate (β) is much greater than the treatment rate (α). Then, after year 60, the $S(t)$ value approaches a constant at around 0.08, indicating that a small portion of the population remains susceptible to infection. This pattern suggests that most of the initially healthy individuals become infected due to the strong transmission force within the population.

2. Infection Compartement (I)

The red curve shown for this compartment shows a significant increase in the proportion of infected individuals at the beginning of the simulation, then peaks around year 50 with a value approaching 0.8. Then, $I(t)$ decreases slowly and plateaus at around 0.79, indicating that the system reaches a stable endemic state. In this phase, the number of infections is balanced by the number of individuals who recover or die. In this steady state, TB remains persistently present in the population rather than being eradicated.

3. Treatment Compartment (T)

As shown in the green curve, which shows a flatter line, the population undergoing treatment is relatively small and not proportionally significant. This indicates that the value of $\alpha = 0.002$ (the rate of individuals entering treatment) is too low compared to ($\beta = 0.18$), so treatment efforts have not been able to effectively reduce the number of infections. Consequently, treatment efforts are insufficient to significantly reduce the number of infected individuals, demonstrating a weakness in disease control and public health intervention within this parameter scenario.

4. Recovery Compartment (R)

As shown in the purple curve, after the 25th year, the number of recovered individuals increases. The proportion of recovered individuals reaches a value approaching 0.07 at the end of the simulation. This figure indicates the small portion of the total population that successfully recovers and gains immunity against reinfection. This limited recovery proportion emphasizes that, without substantial improvement in treatment coverage and prevention measures, the overall impact on reducing TB prevalence remains minimal.

From these results, the basic reproduction number for the parameters used is:

$$\begin{aligned}
 R_0 &= \frac{\theta}{\mu S^{**}} \\
 &= \frac{0,012}{0,013 \times 0,08} \\
 &= \frac{0,012}{0,00104} \\
 &= 11.54
 \end{aligned}$$

Since $R_0 > 1$, then Tuberculosis disease will remain in the population. Therefore, tuberculosis will persist in the population and cannot be eliminated under the current parameter settings. This analytical result aligns with the simulations outcomes, where the number of infective individuals remains high over time and system converges toward an endemic steady state rather than disease extinction.

In conclusion, the simulation clearly illustrates that the high infection rate combined with low treatment effectiveness leads to sustained TB transmission in Aceh Province. To move the system toward disease eradication $R_0 > 1$, public health efforts must focus on increasing treatment rates and preventive measures that effectively reduce transmission.



Philosophical Meaning

This batik motif conveys a profound philosophical reflection on the equilibrium of existence, the harmony that sustains life, and the inseparable connection between all elements of the universe. The flowing ribbons at the center of the motif represent the continuous movement of human life, which never stands still but constantly evolves in rhythm with nature and society. The gentle curves

of these intertwined forms embody the journey of life itself, a journey filled with fluctuations, challenges, and transitions that shape human character and wisdom over time. Life is not a straight or predictable path; it is a flowing current that teaches humans to adapt, endure, and find meaning within transformation.

Each colored ribbon illustrates a different aspect of human experience. The variations of blue, green, red, and purple express diversity in human thought, emotion, and culture. They may also be read as symbols of the many dimensions of human life—intellectual, emotional, spiritual, and moral. Though these colors differ, they flow together into a single, unified rhythm. This unity within diversity becomes the visual and philosophical essence of the motif: the understanding that harmony does not arise from uniformity but from the acceptance of difference. In a broader sense, this reflects a deep moral awareness that peace and coexistence among individuals and communities depend on mutual respect, compassion, and shared purpose.

The floral and leafy ornaments that surround the main pattern strengthen this message of unity. In many traditional philosophies, plants and flowers represent the eternal cycle of life: growth, decay, and regeneration. Here, they serve as reminders that human life is inseparable from the natural world and that both are governed by the same cosmic balance. The leaves reaching outward suggest vitality and hope, while the blooming flowers symbolize the realization of inner potential and the blossoming of virtue. Together they embody the belief that spiritual growth, like natural growth, arises through patience, care, and the harmony between human intention and natural law.

Philosophically, this batik design expresses the conviction that balance in life can only be achieved when all elements—human beings, moral values, and nature—coexist in interdependence. Every curve in the motif reflects a rhythm of existence, a reminder that behind movement there is rest, behind conflict there is reconciliation, and behind every transformation there lies a deeper harmony. The endless flow of the pattern evokes the cyclical nature of the universe, in which beginnings and

endings are not opposites but part of the same eternal process.

Thus, this motif can be seen as a silent meditation on the wisdom of balance. It encourages contemplation of how humans can live in peace with themselves, others, and the world that sustains them. It teaches that harmony is not static but dynamic, continuously renewed through awareness, empathy, and the willingness to adapt. Through this understanding, batik has become more than just a visual art form but also a symbolic expression of human philosophy—a reminder that life, in all its complexity and diversity, is truly beautiful only when done in a way that fits into the larger structure of existence.

Biological Meaning

From a biological perspective, this batik motif embodies the intricate order and interconnectedness that define all living systems in nature. The flowing, wave-like structures at its center evoke the continuous rhythms that sustain life, such as the heartbeat, the circulation of blood, and the movement of air through the lungs. These gentle curves reflect the constant motion within living organisms, where energy and matter are never static but continuously renewed. The motif thus becomes a visual metaphor for the biological principle of homeostasis, which describes the ability of living beings to maintain internal stability while adapting to change. In this sense, every line within the design symbolizes life's resilience, flexibility, and capacity for renewal in the face of environmental shifts.

The leaves and flowers that frame the central composition deepen this biological symbolism. They represent plant life, which serves as the foundation of the Earth's ecosystems and as the planet's main source of energy. Through photosynthesis, plants convert sunlight into food and release oxygen that sustains countless forms of life. The inclusion of these elements in the batik can be understood as a celebration of regeneration, growth, and the self-sustaining cycles of nature. The presence of flowers also conveys ideas of reproduction and continuity, reflecting how plants propagate and ensure the survival of future generations. Together, these elements express the

interdependence that defines the biological world and the importance of maintaining ecological balance to preserve the harmony of life.

The symmetrical arrangement of leaves and flowers highlights another important principle of biology: the natural tendency towards balance and proportion. Symmetry appears across living forms, from the wings of butterflies to the bodies of mammals, and it often serves both aesthetic and functional purposes. In the motif, this mirrored arrangement mirrors the biological order observed in nature. It demonstrates that life achieves stability not through stillness but through structured organization and repetition. The repetition of forms within the pattern mirrors the repetition of genetic codes and cellular processes that maintain the continuity of life across generations.

At the same time, the interaction of colors, shapes, and lines within the motif can be interpreted as a visual representation of an ecological system. Each component contributes to the whole, much like organisms within a habitat rely on one another for survival. No element in the motif stands alone. Energy and matter flow among the components, sustaining the unity of the design just as nutrients circulate among organisms in a natural environment. This interconnectedness reminds the observer that the health of one part of the system influences the well-being of the rest. The batik, therefore, becomes a symbolic expression of the delicate balance that exists within ecosystems, where cooperation and adaptation ensure the persistence of life.

Ultimately, this motif offers a powerful reminder that humans are part of the same biological web that sustains every form of existence. The intertwining ribbons and organic ornaments emphasize that life's beauty emerges when harmony is achieved between human activity and natural processes. The motif encourages awareness of the responsibility humans carry to protect and restore the environment that nurtures all living beings. In this way, the batik is more than a work of art. It becomes a reflection of biological wisdom, showing that the patterns of life, when observed closely, always point back to

balance, renewal, and the deep unity of all living things.

Mathematical Meaning

From a mathematical standpoint, this batik motif reveals a deep appreciation for order, structure, and logic. The repeating wave-like ribbons display a rhythm that can be interpreted as a visual expression of periodic mathematical functions. The rise and fall of the curves resemble the behavior of sinusoidal waves, which oscillate in continuous and balanced cycles. This repetition conveys the idea that beauty in mathematics often emerges from regularity and predictability. The motif thus becomes a symbolic illustration of dynamic equilibrium, where change unfolds within boundaries that remain stable over time. Through its visual form, the pattern demonstrates how movement and balance coexist in harmony, echoing the mathematical principle that stability can arise from continuous variation.

The symmetry evident in the motif reinforces the importance of geometric reflection, one of the fundamental concepts in mathematical design. Each side of the motif mirrors the other, creating a visual sense of harmony that mirrors the precision of geometric transformations. Symmetry in mathematics represents balance and consistency, and its presence in this batik reinforces the idea that aesthetic order often emerges from formal rules. This kind of bilateral symmetry also serves as a metaphor for mathematical equality, where every element on one side has a corresponding counterpart on the other. By aligning the design so precisely, the batik communicates the same sense of proportion and coherence that mathematicians seek in equations, proofs, and models.

The small dotted ornaments that trace the surface of the ribbons introduce another mathematical dimension. These dots can be seen as representations of discrete points in an otherwise continuous space, similar to the way numerical models approximate real phenomena. In numerical analysis, continuous systems are often broken down into smaller units to make them more understandable and measurable. Likewise, the dotted texture in the motif transforms the flowing wave into something quantifiable, echoing the

process of discretization that lies at the core of modern computation and applied mathematics. This combination of smoothness and granularity mirrors how mathematics bridges the gap between idealized theory and practical representation.

Beyond its aesthetic qualities, the motif also embodies ideas central to applied mathematical modeling. The three colored ribbons may be understood as visual analogies of interrelated variables within a system, such as the compartments of an epidemiological model like SITR, which stands for Susceptible, Infective, Treatment, and Recovery. Each ribbon interacts with the others just as the variables in such models interact over time. The intersections represent the dynamic exchanges that occur between states in a population, while the flowing motion of the lines reflects the temporal evolution toward equilibrium. Through this interpretation, the motif becomes a metaphor for the mathematical study of systems that seek balance through constant interaction and transformation.

The batik design expresses a profound connection between mathematical reasoning and natural beauty. The repetition of shapes, the precision of symmetry, and the underlying sense of rhythm all reveal how the world operates according to mathematical principles. Yet within this structure lies creativity and imagination, reminding us that mathematics is not limited to abstraction but also serves as a language for interpreting reality. The motif captures the spirit of mathematical thought, demonstrating that behind every elegant pattern there is a logic that unites complexity with clarity. In this way, art and mathematics converge as complementary ways of understanding the same truth: that order and beauty are inseparable within the fabric of the universe.

Conclusions

This study has successfully formulated, simulated, and analyzed the SITR (Susceptible–Infective–Treatment–Recovery) mathematical model to describe the transmission dynamics of tuberculosis (TB) in Aceh Province. By constructing a system of

nonlinear differential equations and solving it numerically using MATLAB's ode45 solver, the research was able to capture the continuous and interdependent processes governing the spread, treatment, and recovery of TB within a population.

The stability analysis of the model revealed the existence of two equilibrium points: a disease-free equilibrium and an endemic equilibrium. The disease-free equilibrium represents the ideal condition in which TB is eradicated from the population, and it is mathematically stable only when the basic reproduction number (R_0) is less than one. Conversely, when $R_0 > 1$, the endemic equilibrium becomes stable, indicating that TB remains persistent within the community. Based on the parameters used in this study—particularly the relatively high infection rate ($\beta = 0.18$) and low treatment rate ($\alpha = 0.002$)—the computed R_0 value was 11.54, which strongly suggests that TB transmission in Aceh will continue unless significant interventions are introduced to reduce new infections or enhance treatment efficiency.

The numerical simulation results supported this theoretical finding. Over the simulated time period, the susceptible population showed a rapid decline as more individuals became infected. The infective population increased sharply and then stabilized at a high proportion, indicating an endemic steady state. Meanwhile, the treatment compartment remained relatively small, reflecting the limited capacity of treatment programs to offset the high infection pressure. Although the recovery rate ($\gamma = 1.5$) was substantial, its impact was not sufficient to achieve disease elimination, as only a small portion of the total population successfully transitioned to immunity.

These results underscore the importance of public health strategies focused on improving early detection, expanding access to effective treatment, and strengthening community-based prevention programs. In particular, increasing the treatment rate (α) and reducing the infection rate (β) through sustained public awareness, hygiene improvement, and health system reinforcement could help bring R_0 below unity and move the population toward a disease-free equilibrium.

Beyond its epidemiological implications, this study also introduced an innovative integration of

mathematical modeling with cultural art through batik visualization. The Sitr-based batik pattern designed in this research serves not only as a visual metaphor for the cyclic processes of infection, treatment, and recovery but also as a creative medium for translating complex mathematical insights into accessible cultural expressions. The repeating motifs and symmetrical elements of the batik design symbolize the delicate balance between disease spread and human efforts to restore health—a concept deeply resonant with the values of harmony and resilience in Acehese culture.

Through this interdisciplinary approach, the study demonstrates that mathematics and art can coexist as complementary tools for understanding and communicating public health phenomena. While the Sitr model provides a rigorous quantitative framework for analyzing disease dynamics, the batik visualization extends its reach, allowing scientific knowledge to be appreciated in an aesthetic and culturally meaningful form.

In summary, this research not only contributes to the scientific understanding of tuberculosis transmission dynamics in Aceh Province but also pioneers a novel way of linking epidemiological modeling with cultural expression. It highlights the potential for mathematical models to inform real-world health strategies while fostering broader public engagement through creative interpretation. Thus, the Sitr model—enriched by its batik-based representation—serves as both a scientific and cultural bridge, embodying the ongoing dialogue between knowledge, tradition, and human resilience in confronting infectious diseases.

Acknowledgements: Thanks to Batik Ghakhuka, Address: Mredo, Bangunharjo, Sewon, Bantul, Special Region of Yogyakarta, Indonesia, which is used as a place for batik making.

Conflict of Interest: The authors declare that there are no conflicts of interest concerning the publication of this article.

References

- Amatillah, D. I., Ilahi, F., & Khumaeroh, M. S. (2022). *Analisis sensitivitas dan kestabilan global model pengendalian tuberkulosis dengan vaksinasi, latensi, dan perawatan infeksi*. KUBIK: Jurnal Publikasi Ilmiah Matematika, 7(2), 83–94. <https://journal.uinsgd.ac.id/index.php/kubik/article/view/14938>
- Badan Pusat Statistik Provinsi Aceh. (2021). *Profil kesehatan Provinsi Aceh tahun 2021*. BPS Provinsi Aceh.
- Roudhotillah, D., & Chandra, T. D. (2022). *Analisis kestabilan model penyebaran penyakit tuberkulosis dengan menggunakan MSEITR*. Jurnal Pendidikan Matematika, 13(1), 23–31. <https://ejournal.undiksha.ac.id/index.php/JPM/article/view/34503>
- Simorangkir, G., Aldila, D., Rizka, A., Tasman, H., & Nugraha, E. S. (2021). *Mathematical model of tuberculosis considering observed treatment and vaccination interventions*. Journal of Mathematical and Computational Science, 11(6), 6937–6955. <https://doi.org/10.1080/09720502.2021.1958515>
- Suryawan, D. A., & Siagian, T. H. (2021). *Sex and age group differences in the spread of tuberculosis in Indonesia: An agent-based modeling approach*. Jurnal Aplikasi Statistika dan Komputasi Statistik, 13(2), 133–142. <https://doi.org/10.34123/jurnalasks.v13i2.342>
- World Health Organization. (2024). *Global tuberculosis report 2024*. Geneva: WHO Press.
- Zahwa, N., Nabilla, U., & Nurviana, N. (2022). *Model matematika SITR pada penyebaran penyakit tuberkulosis di Provinsi Aceh*. Jurnal Pendidikan Matematika dan Sains (JPMS), 10(2), 145–153. <https://jurnal.uny.ac.id/index.php/jpms/article/view/50683>