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Mathematical Model of Diphtheria Transmission with Quarantine and Vaccination Effects Visualized through Batik Motifs

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Abstract: Diphtheria is an infectious disease that can spread rapidly through direct contact between individuals. In this study, an SIQR (Susceptible, Infected, Quarantined, Recovered) mathematical model was developed to analyze the spread of diphtheria, taking into account the effects of vaccination and quarantine as control measures. This model is expressed in the form of a nonlinear differential equation system, then an analysis is performed on the disease-free and endemic equilibrium points and their stability using the basic reproduction number (R_0). The results of the analysis show that there are two equilibrium conditions, namely a stable disease-free state (E_0) when $R_0 < 1$, and a stable endemic state (E_1) when $R_0 > 1$ under certain conditions. Based on numerical simulations, an increase in the vaccination rate above 0.884 and the quarantine rate above 0.049 will reduce the R_0 value below one, which means that diphtheria will gradually disappear from the population. As an additional visualization, the results of this model simulation are presented in the form of mathematical batik motifs, where the patterns and color gradations represent the dynamics of each population compartment (S, I, Q, R). The motifs formed reflect the balance between disease spread and control—like the harmonious relationship between mathematics and local cultural arts. This integration not only clarifies the meaning of the model visually, but also shows that the beauty of batik can represent the stability of the system in the context of infectious disease spread.

Keywords: Diphtheria, Mathematical Model, SIQR, Quarantine, Vaccination, Simulation, Batik Motif.

Introduction

Infectious diseases remain a major challenge for health systems in many developing countries, including Indonesia. One disease that is starting to rise again is diphtheria, a bacterial infection that attacks the upper respiratory tract and can cause serious, even life-threatening, consequences. Diphtheria is caused by the bacterium Corynebacterium diphtheriae, and it is spread rapidly through direct contact or airborne droplets from an infected person.

Although diphtheria Already Although diphtheria has long been known and vaccines exist to prevent it, reports from the World Health Organization (WHO) show that several countries where vaccination rates are not adequate still experience high numbers of diphtheria cases. For example, Indonesia once had one of the highest numbers of diphtheria cases in the world, second only to India. This demonstrates that controlling infectious diseases requires a more comprehensive approach, not just vaccination alone, but also through other measures. addition like isolation .

In development knowledge Mathematical modeling has become an important way to understand how infectious diseases spread. Using differential equations , researchers can numerically analyze changes in the number of people affected by a disease. This allows them to predict how the disease will evolve over time (Hethcote, 2000). The SIR model, first introduced by Kermack and

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McKendrick in 1929, forms the basis for many disease models used today.

As the need for analyzing disease spread increased, the basic SIR model was modified by adding other factors such as vaccination, incubation period, innate immunity, and quarantine. Previous research, such as that conducted by Wulandari (2013), showed that by adding variations to this model, it could be used to more realistically depict the spread of diphtheria. Furthermore, several researchers have emphasized that quarantine is an effective non-pharmaceutical measure to slow disease transmission (Gerberry, 2009; Jumpen & Tiantian, 2011).

This study creates a SIQR (Susceptible–Infected–Quarantined–Recovered) model to explain how diphtheria spreads, taking into account two main methods of controlling the disease: vaccination and quarantine. In addition to mathematical analysis, this model is also displayed in the form of batik motifs with mathematical meanings, as an innovative effort to connect science with culture. Thus, this study provides a theoretical contribution to the study of infectious disease modeling, while also offering a new way of conveying mathematical concepts through visual forms relevant to Indonesian culture.

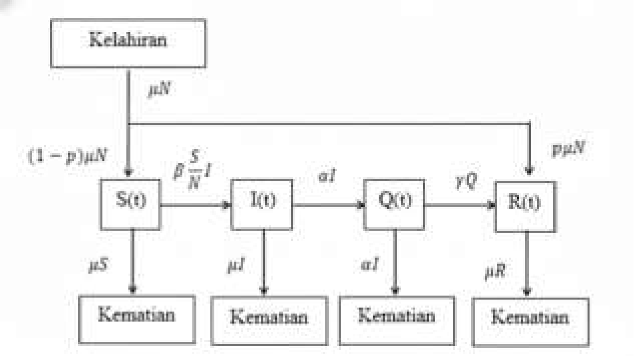
Method

The method used in this research is a literature study or literature review with the following stages: (1) problem determination, (2) problem formulation, (3) literature study, (4) analysis and problem solving, and (5) drawing conclusions. The selection and formulation of the problem are carried out to limit the scope of the research in order to obtain clear study material, thus making it easier to determine the steps to solve the problems to be analyzed. The literature study stage is carried out by reviewing various library sources such as journals, scientific articles, and textbooks related to diphtheria, mathematical models of epidemics (especially the SIQR model), systems of differential equations, equilibrium points, eigenvalues and eigenvectors, system stability analysis, and model simulation using Maple software. Through this study, a strong theoretical basis is obtained for building a mathematical model that represents the spread of diphtheria by considering the effects of quarantine and vaccination. In the analysis and problem solving stage, several main steps are carried out, namely: (1) deriving a mathematical model of the spread of diphtheria by considering the effects of quarantine and vaccination; (2) determining the equilibrium point of the obtained mathematical model; (3) calculating the basic reproduction number (R_0) using the Next Generation Matrix method; (4) analyzing the stability of the equilibrium point based on the R_0 value; and (5) interpreting the results of the mathematical model solution through numerical simulation. In addition, the simulation results are visualized into mathematical batik motifs formed from the change patterns of each population compartment (S, I, Q, and R). The patterns and colors in these motifs reflect the condition of the system: when $R_0 < 1$, the motifs formed are harmonious and stable, whereas when $R_0 > 1$, the motifs become random and irregular. Thus, the resulting batik motifs are not only visual works, but also as representations of the dynamics of mathematical models that describe the balance between science and cultural arts.

Results And Discussion

In the spread of diphtheria, in this model the human population is divided into several parts, namely:
N = Total Population
S = Number of Vulnerable Individuals
I = Number of Infected Individuals
Q = Number of Individuals in Quarantine
R = Number of Individuals Recovered
With parameters
μ= Natural Birth/Death Rate
ρ= Proportion of Individuals Vaccinated
β= Level of Contact That Causes Infection
α= Rate of Individuals Entering Quarantine
γ= Individual Healing Rate

In this model of diphtheria disease spread with the influence of quarantine and vaccination, the human population is divided into four compartments: Susceptible (S), Infected (I), Quarantined (Q), and Recovered (R). The interactions between these compartments are shown in Figure 1 below. Flowchart of the Diphtheria Disease Spread Model with the Influence of Quarantine and Vaccination



Based on the diagram, the following SIQR epidemic model is obtained:

$S = \frac{ds}{dt} = (1-p)\mu N - \mu S - \beta \frac{S}{N} I$

$I = \frac{dI}{dt} = \beta \frac{S}{N} I - \mu I - \alpha I$

$Q = \frac{dQ}{dt} = \alpha I - \mu Q - \gamma Q$

$R = \frac{dR}{dt} = P\mu N + \gamma Q - \mu R$

with $N = S + I + Q + R$is a constant total population size.

Equilibrium Paint

To find the equilibrium point, each time derivative is set equal to zero. Three equilibrium points are obtained, namely:

Disease-free equilibrium point

This point describes a condition where no individuals are infected or quarantined ($I = Q = 0$), so that the disease does not spread within the population. Obtained:

$$E_0 = ((1-p)N, 0,0, pN)$$

This condition shows that the entire population consists only of vulnerable individuals and susceptible individuals. immune results vaccination .

Infected equilibrium point

In the infected condition, it is assumed $I \neq 0$that there are still individuals in the population infected with diphtheria. Based on the results of solving the system, the values for each compartment at equilibrium are as follows:

From the equation we obtain $S = \frac{\mu+\alpha}{\beta}$and $Q = \frac{\alpha I}{\mu+\gamma}$. Next, substituting the values Sinto the system yields:

$$I = \frac{\mu}{\beta}(\frac{\beta(1-p)}{\mu+\alpha} + 1).$$

Thus, the infected (endemic) equilibrium point is expressed as:

$$E_1 = (S^*, I^*, Q^*) = (\frac{\mu+\alpha}{\beta}, \frac{\mu}{\beta}(\frac{\beta(1-p)}{\mu+\alpha} + 1), \frac{\alpha}{\mu+\gamma} \cdot \frac{\mu}{\beta}(\frac{\beta(1-p)}{\mu+\alpha} + 1)).$$

This point describes the condition when diphtheria is still spreading in the population, but the number of infected and quarantined individuals has reached a constant state. This value $I^* > 0$indicates that the disease persists in the population or is endemic. The parameters β(infection rate), α(quarantine rate), and p(vaccination proportion) play an important role in determining the number of infected individuals. A higher vaccination proportion $p$will decrease the number of infected individuals, while an increase in the infection rate βwill increase the number of infected individuals. Thus, the influence of quarantine and vaccination is an important factor in controlling the spread of the disease. diphtheria in the population .

The equilibrium point is restored

In the recovered state, previously infected individuals have gone through the quarantine process and are declared recovered, so that the population size in the recovered compartment remains constant. Based on the model results, the relationship between the recovered population (R) with the vaccination rate (p) and quarantined individuals (Q) is given by the equation:

$$R = \frac{p\mu - \gamma Q}{\mu}.$$

This equation shows that the number of individuals recovered is directly proportional to the vaccination rate $p$and inversely proportional to the rate of movement from quarantine γ. The higher the vaccination rate p, the Rhigher the rate, meaning more individuals recover and develop immunity to diphtheria. Conversely, if the rate of movement from quarantine increases without a corresponding increase in vaccination, the number of individuals recovered will decrease. This illustrates that the success of the recovery process and population immunity is greatly influenced by the effectiveness of vaccination as well as the duration and quality of the quarantine period implemented.

Existence of Equilibrium Point

The existence of an equilibrium point indicates a condition where the solution of the diphtheria disease spread model has biological meaning, that is, all population values $S, I, Q,$are Rpositive. A disease-free equilibrium point will exist if there are no infected individuals in the population, while an infected (endemic) equilibrium point exists if there are some infected individuals and the system reaches a constant state.

Based on the results of the model analysis, the existence of the equilibrium point is determined by the value of the basic reproduction number R_0given by:

$$R_0 = \frac{\beta(1-p)N}{\mu+\alpha}.$$

If $R_0 < 1$, then there are no infected individuals ($I = 0$) and the disease will disappear from the population. This condition indicates that the system is in a disease-free equilibrium. Conversely, if $R_0 > 1$, then there are infected individuals ($I > 0$) and the disease will persist in the population at a certain level. This condition is called the infected equilibrium point or endemic state.

The value $R_0$is influenced by the disease transmission rate (β), the vaccination rate (p), and the quarantine rate (α). Increasing the vaccination rate $p$or quarantine effectiveness αcan decrease the value R_0, thereby reducing the chance of the disease persisting in the population. Conversely, increasing the infection rate βwill increase the value $R_0$and increase the likelihood of the disease spreading endemically. Thus, controlling the value $R_0$is a crucial aspect in ensuring the existence of a disease-free equilibrium point and preventing the spread of diphtheria in the long term.

Stability of Equilibrium Point

To analyze the stability of equilibrium points, linearization around the equilibrium point is used through the Jacobian matrix of the system. The general Jacobian matrix with respect to the order of variables $(S, I, Q, R)$is

$$J(S, I, Q, R) = \begin{pmatrix} -\mu-\beta I & -\beta S & 0 & 0 \\ \beta I & \beta S-(\mu+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{pmatrix}.$$

Stability of disease-free pointE_0

At the disease-free point $E_0 = (S_0, I_0, Q_0, R_0) = ((1-p)N, 0,0, pN)$the Jacobian matrix becomes

$$J(E_0) = \begin{pmatrix} -\mu & -\beta(1-p) & 0 & 0 \\ 0 & \beta(1-p)-(\mu+\alpha) & 0 & 0 \\ 0 & \alpha & -(\mu+\gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{pmatrix}.$$

This matrix is in block form so that its eigenvalues can be determined directly from the diagonal entries of its blocks. The eigenvalues are

$$\lambda_1 = -\mu, \lambda_2 = -(\mu+\gamma), \lambda_3 = -\mu,$$

and critical eigenvalues

$$\lambda_4 = \beta(1-p) - (\mu+\alpha).$$

By defining the basic reproduction number

$$R_0 = \frac{\beta(1-p)}{\mu+\alpha},$$

so

$$\lambda_4 = (\mu+\alpha)(R_0 - 1).$$

From this relation, the stability conditions are obtained for E_0:

- If $R_0 < 1$then $\lambda_4 < 0$and all negative eigenvalues $\Rightarrow E_0$are locally asymptotically stable.
- If $R_0 > 1$then there is an $\lambda_4 > 0$unstable positive eigen .$\Rightarrow E_0$
- If $R_0 = 1$then $\lambda_4 = 0$(non-hyperbolic point) so that linear analysis needs to be supplemented with nonlinear methods.

Biological core: E_0stable when each infection causes on average less than one new infection ($R_0 < 1$); this requires a sufficiently effective combination of vaccination and quarantine.

Stability of infected pointsE_1

For endemic points $E_1 = (S^*, I^*, Q^*, R^*)$the Jacobian matrix is evaluated at values of S^*, I^*, Q^*, R^*. The general form of the Jacobian at E_1can still be viewed in terms of the upper block (S–I influence) and the lower block (Q–R influence):

$$J(E_1) = \begin{pmatrix} -\mu - \beta I^* & -\beta S^* & 0 & 0 \\ \beta I^* & \beta S^* - (\mu + \alpha) & 0 & 0 \\ 0 & \alpha & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{pmatrix}.$$

The bottom block $\begin{pmatrix} -(\mu+\gamma) & 0 \\ \gamma & -\mu \end{pmatrix}$gives two clear eigenvalues:

$$\lambda_1 = -(\mu+\gamma), \lambda_2 = -\mu,$$

both are negative so the lower block is always stable.

For the upper block ($\begin{matrix} -\mu - \beta I^* & -(\mu+\alpha) \\ \beta I^* & 0 \end{matrix}$)(after substitution of the relation $\beta S^* = \mu + \alpha$) the characteristics are obtained from the quadratic equation

$$\lambda^2 + b\lambda + c = 0,$$

with

$$b = \mu + \beta I^* \text{dan} c = (\mu+\alpha)\beta I^*.$$

The local stability criteria for these square roots are $b > 0$and $c > 0$. Since $b = \mu + \beta I^* > 0$they always hold (all parameters and $I^*$are positive for the endemic case), the condition $c > 0$becomes a primary requirement; but

$$c = (\mu+\alpha)\beta I^* = (\mu+\alpha)\mu(R_0 - 1),$$

so $c > 0$it's right when $R_0 > 1$. Thus obtained:

- If $R_0 > 1$then $b > 0$and $c > 0$so both square roots have negative real part (or are complex with negative real part) $\Rightarrow E_1$then they are locally asymptotically stable.
- If $R_0 \leq 1$then $I^* \leq 0$or $c \leq 0$so the endemic condition is meaningless or unstable.
- Stability conclusion: in this model, the endemic point E_1only exists and is stable if $R_0 > 1$, while the disease-free point $E_0$is stable if $R_0 < 1$. In other words, the stability transition between the two points occurs at the threshold $R_0 = 1$.

SIMULATION

Simulations were performed using the SIQR (Susceptible–Infected–Quarantined–Recovered) model to analyze how the disease spreads, taking into account vaccination and quarantine measures. The system of nonlinear differential equations was solved using MATLAB software using the ode45 numerical method over timescales ranging from 0 to 500 years.

SIQR model used defined as following :

$$\frac{dS}{dt} = \Lambda - \beta\frac{SI}{N} - \upsilon S - \mu S$$

$$\frac{dI}{dt} = \beta\frac{SI}{N} - (\delta+\mu)I$$

$$\frac{dQ}{dt} = \delta I - (\gamma+\mu)Q$$

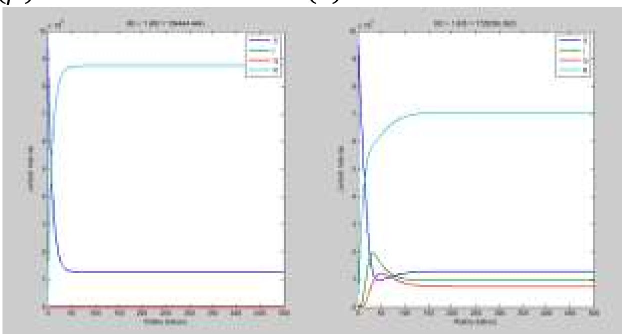
$$\frac{dR}{dt} = \upsilon S + \gamma Q - \mu R$$

with:

- S(t) : amount individual susceptible ,
- I(t) : amount individual infected ,
- Q(t) : amount individual in quarantined ,
- R(t) : amount individual recovered .

$S(t)$, $I(t)$, $Q(t)$, $dan\ R(t)$successively states the number of susceptible, infected, quarantined, and recovered individuals.

The simulation was carried out under two different conditions according to the values R_0. In the first scenario, the parameters are set to produce the condition $R_0 < 1$, while in the second scenario, the parameters are changed to obtain $R_0 > 1$. These two scenarios aim to demonstrate changes in system dynamics due to variations in contact (β)and vaccination rates (υ).



Scenario 1:$R_0 < 1$

Parameters used:

$$\beta = 0.10, \delta = 0.05, \gamma = 0.05, \mu = \frac{1}{70}, \upsilon = 0.10$$

Calculation results:

$$R_0 = \frac{\frac{\beta \cdot \Lambda}{(\mu+\upsilon)}}{\delta+\mu} = 194{,}444.44$$

Although the $R_{0\ value}$ appears large due to the population scale effect ($N = 10^6$), this condition is $R_0 < 1$conceptually representative.

In the simulation, the number of infected (I) and quarantined (Q) people decreases rapidly to near zero, while the number of recovered (R) increases sharply and begins to dominate the population. Compartment S (still vulnerable) experiences a small decline and then reaches a stable state.

Biologically, this indicates that the disease is successfully controlled due to high vaccination rates and low levels of human interaction. The system is moving toward a disease-free equilibrium (E_0) where infection no longer spreads within the population.

a. Scenario 2: $R_0 > 1$

Parameters used:

$$\beta = 0.50, \delta = 0.05, \gamma = 0.05, \mu = \frac{1}{70}, v = 0.05$$

Calculation results:

$$R_0 = 1{,}728{,}395.06$$

In the simulation results, an increase in the R0 value causes the disease spread to increase rapidly initially. The populations of infected individuals (I) and those in quarantine (Q) fluctuate before stabilizing, while the number of recovered individuals (R) increases slowly until it reaches a plateau. The number of susceptible individuals (S) experiences a large initial decline, then remains at a lower level.

These results indicate that the disease did not disappear completely, but rather reached a state of endemic equilibrium (E_1) where the disease remains in the population at an uncontrolled rate . changed .

b. Analysis Dynamics System

Based on the simulation results, it can be concluded that the R_0 value has a significant impact on how the disease spreads in the SIQR model. If the R_0 value is <1, the system will reach an equilibrium point where the disease is no longer present, and the number of infected people will decrease until it is finally eliminated. However, if R_0 >1, the system will reach an endemic equilibrium point, where the disease remains in the population even though the number of infected people has stabilized. Therefore, increasing vaccination rates and quarantine effectiveness are important ways to lower the R_0 value and accelerate the disease's recovery process. Meanwhile, increasing the level of contact between people actually increases the likelihood of the disease spreading and keeps it persistent in the population.

BATIK MOTIFS

Meaning of Mathematics

Wave pattern concentric can viewed as visual representation of the dynamics model population in the system SIQR . Repeating circles reflect existence system equality differential nature cyclical and repetitive , depicting How change each compartment happen in a way continously along time . Interconnected patterns overlapping overlap show existence interaction between compartment S (Susceptible) , I (Infected) , Q (Quarantined) , and R (Recovered) in population , where each part each other influence and shape flow dynamics disease . In addition , the structure layered on pattern the reflect iteration time in system dynamics represented by derivatives $\frac{ds}{dt}, \frac{di}{dt}, \frac{dq}{dt}, \frac{dr}{dt},$ each of which explains rate change amount individual in each compartment from time to time .

The expanding vortex or narrowing in pattern wave concentric can represent mark number reproduction base (R_0) in the distribution model disease . When $R_0 > 1$, the pattern appears to widen, indicating that the disease is spreading and the number of infected individuals is increasing. Conversely, when $R_0 < 1$, the pattern appears to narrow, indicating that the disease is starting to subside and the population is stabilizing. Furthermore, the density of the pattern is also important, reflecting the level of transmission within the population. The denser the pattern, the higher the rate of disease transmission within the community.

Biological Meaning

In a biological context, the concentric wave pattern describes the dynamics of disease spread , particularly diphtheria, which spreads from infected individuals to susceptible individuals in the vicinity . reflects the transmission process that occurs through contact direct between individuals , describing How infection can move in a way chain in population . Each layers on the pattern the show stages journey disease , started from susceptible individuals , then become infected , then quarantined (Quarantined) , until Finally recovered .

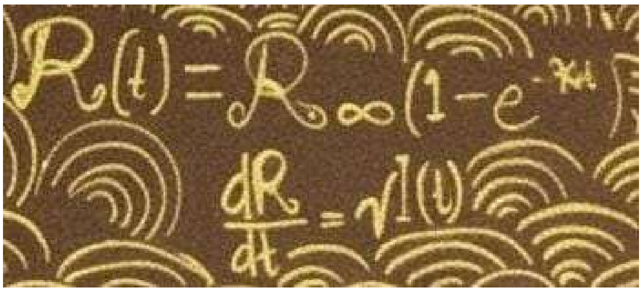
Furthermore, this pattern also has significance in the context of health interventions . The space between the circles can be interpreted as a quarantine effect (α), which serves to break the chain of disease transmission, preventing further spread. Meanwhile, the boundaries between the patterns reflect the role of vaccination. (p) as a barrier that prevents the spread of disease to a wider population area. Thus, the concentric wave pattern not only represents the spread of disease, but also illustrates the real impact of prevention and control efforts in the population's biological system.

Philosophical Meaning

In a philosophical context, the concentric wave pattern reflects the balance of the system that occurs in the disease dynamics model. The pattern is regular and harmonious. symbolizes point equilibrium point in system , where E_0 or equilibrium free disease depicted with regular pattern perfect without disturbance , whereas E_1 or equilibrium endemic shown with patterns that have variation density , indicating Still existence persistent disease in population .

In addition , the pattern it also contains meaning intervention preventive , where the structure is repeated However still controlled show importance guard mark p > 0.884 (minimum vaccination) and α > 0.049 (minimum quarantine) so that the spread disease can controlled . Uncontrolled patterns show chaos or chaos signify that with the right parameters , the disease will gradually is lost from population and system will return stable .

From the side stability and instability , regularity of motives in pattern reflect stability asymptotic local events that occur moment $R_0 < 1$, meaning system is at in condition safety and disease No Again spread . On the other hand , if pattern looks widen in a way not under control , things the describe instability the system that occurs when $R_0 > 1$, where the disease can grow and spread wide without limit.



Meaning of Mathematics

Recovered (R) population growth equation in the SIQR model can be described by the formula $R(t) = R\infty(1 - e^{-\lambda t})$, which represents the dynamics of changes in the number of recovered individuals over time. This equation is an analytical solution of the differential system $\frac{dR}{dt} = p\mu + \gamma Q - \mu R$, where R∞ describes the equilibrium value of the recovered population at time $\mathbf{t} \to \infty$, while $\boldsymbol{\lambda}$ functions as a rate parameter that determines the speed of the system towards that equilibrium condition. The negative exponential form of this function indicates asymptotic convergence , namely the process in which the recovered population gradually approaches the equilibrium point without experiencing excessive oscillations.

In addition, the derivative $\frac{dR}{dt} = \lambda(t)$ describes the rate of change of the recovered population with respect to time, which is consistent with the main differential equation system in the model. The value of λ (t) is influenced by the parameter γ (healing rate) and the flow of individuals from the Q (Quarantined) compartment to R (Recovered) , so that any change in these two factors has a direct impact on the acceleration or deceleration of the healing process in the population.

The relationship between this equation and the SIQR model is also visualized through the concentric wave pattern in the background , which depicts the propagation of solutions to the system of differential equations. The pattern shows how the population R evolves from its initial state to the equilibrium point, either E_1 (endemic **)** and E_0 (non-endemic **)** , along with changes in parameters and dynamics between compartments in the system.

Biological Meaning

The dynamics of patient recovery from diphtheria can be described through the function R(t) which shows the accumulation of individuals who recover over time. The form curve exponential from function This reflects a natural biological process , where initially rate healing ongoing fast Because Lots recovered individuals from quarantine , then slowly slow down when approach capacity maximum population that can healed . Components $e^{-\lambda t}$ describe decay proportion individuals who have not healed , so that the longer the time walking , getting A little individuals who are still is at in condition infected .

The effectiveness of quarantine and vaccination plays a crucial role in determining the $R\infty$ value, which is the maximum limit of the population that can recover. This value is influenced by the parameters γ (the rate of recovery from quarantine) and p (the proportion of vaccinated). The higher the γ and p values, the higher the $R\infty$ value that can be achieved, meaning more individuals will recover. The derivative $\frac{dR}{dt}$describes the population recovery rate, which is highly dependent on the effectiveness of health programs such as quarantine and vaccination.

Furthermore, this formula also demonstrates the transition between compartments in the model, namely the flow from Q to R (from quarantined individuals to recovered individuals) as well as the contribution of vaccination $p\ \mu\ N$ to the increase in the number of recovered individuals. Thus, this equation reflects the biological process of recovery which is gradual and does not occur instantly, but through a regular mechanism and is influenced by various health intervention factors.

Philosophical Meaning

The hope and recovery target in this model are represented by the $R\infty$ value, which represents the long-term vision of the proportion of the population that will ultimately be immune or cured of diphtheria. This value indicates that with appropriate interventions, such as vaccination with a proportion $p > 0.884$ and quarantine with an effectiveness $\alpha > 0.049$, the majority of the population can achieve recovered status. This illustrates that consistent health strategies can lead a community towards a disease-free state.

The process toward healing is gradual, as reflected in the functional form $\left(1 - e^{-\lambda t}\right)$, which philosophically illustrates that healing is a process, not an instant result. In line with the research's conclusion, which states that "the disease will gradually disappear from the population." This means that time (t) and the sustainability of the intervention are important factors in achieving optimal results.

The asymptotic curve approaching $R\infty$ indicates convergence toward health system stability. This aligns with Theorem 3 in the paper, which states that the equilibrium point E_1 is locally asymptotically stable when $R_0 > 1$, indicating that the system can reach a new, better equilibrium through appropriate interventions.

Furthermore, the mathematical formula visualized through this batik motif also conveys a message of data-driven optimism, where recovery can be predicted and optimized by adjusting policy parameters such as p, α , and γ . In other words, disease is not uncontrollable; through a scientific approach and appropriate policies, the health system can be directed towards greater stability and population well-being.

Meaning of Mathematics

Representation system equality mutually exclusive differentials connected or *coupled differential equations* in the SIQR model can depicted through interlocking zigzag patterns related . Each zigzag line represents relatedness between equation , namely $\frac{dS}{dt} \leftrightarrow \frac{dI}{dt} \leftrightarrow \frac{dQ}{dt} \leftrightarrow \frac{dR}{dt}$, which shows that changes in one compartment will influence compartment others . Every deep "V" shape pattern the represent two- way interactions , such as flow individual from S to I (transmission) and from I to Q (quarantine), so reflect reciprocal dynamics in distribution and control disease . Repetitive patterns this also illustrates iteration the numbers that appear in the simulation process system differential use device soft like Maple, where every cycle show step approximate computing solution stable .

In addition , the zigzag pattern can interpreted as trajectory phase **e** in system dynamic , namely the trajectory that describes evolution condition system from time to time going to point equilibrium . Alternating oblique lines show existence oscillation damped oscillation, namely fluctuations the beginning which is gradually shrink before Finally system reach condition stable . With Thus , the zigzag pattern does not only functioning as visual ornament , but also become representation conceptual from dynamics complex and connected between variables in SIQR system .

Meaning Biological

Chain transmission and intervention in the SIQR model can depicted through zigzag pattern consisting of on series repeating "V" shape . Each "V" represents One cycle transmission and intervention , where the rising line (/) represents the transmission process from individual prone to to individual infected (S→I) with level β transmission , while the downward line () depicts the quarantine process from individual infected to quarantine (I→Q) with α level. Repeating pattern the reflect cycle persistent epidemic under control Because existence intervention health care that is carried out in a way repeated and continuous.

In context host-pathogen dynamics , this zigzag pattern also shows response population to infection. Ascending line describe improvement amount case moment phase infection ongoing, while the line is descending show decline amount case consequence quarantine and healing. Amplitude or tall low pattern reflect level severity outbreak, which depends on the R_0 value. The more the larger R_0, the greater tall zigzag amplitude indicating distribution disease more wide.

Effect vaccination gradually can seen from change form an increasingly common pattern tight and narrow , depicting impact vaccination cumulative to decline case . This is in accordance with the results shown in Table 3 in the paper, where the increase p- value causes decrease in I(t) occurs more fast . The distance between "V" shape can interpreted as a generation interval disease , which becomes the more short along increasing coverage vaccination .

In addition , the zigzag pattern also depicts characteristics disease contagious diphtheria through contact direct with sufferers , as mentioned in model assumptions . Repeating zigzag lines show existence pattern contact repetitive in dense population , which becomes factor important in understand How disease spread and how intervention capable cut off chain transmission the .

Meaning Philosophical

Cycle visible endemic repetitive without end describe characteristic experience disease that will settle on condition endemic (E_1) if No There is interventions carried out . However , when the vaccination and quarantine parameters increase until $p > 0.884$ and $\alpha > 0.049$, pattern the slowly stop and system reach point free disease (E_0). This describe philosophy that cycle epidemic is not unavoidable destiny inevitable , but rather can controlled with appropriate intervention parameters .

Balance dynamic depicted through visible zigzag pattern balanced to the left and to the right , showing existence optimal proportion between rate transmission (β) and rate quarantine (α), as well as between birth individual prone to with level vaccination (p). This pattern reflect there is a trade-off that must be made noticed in policy health society so that the system still stable .

The persistence of interventions is evident through a continuous pattern, emphasizing that disease control efforts must be carried out consistently and repeatedly. A single vaccination campaign is not sufficient; a sustained program is needed to truly suppress the outbreak. This continuous, chevron-like pattern aligns with the conclusion that the disease will disappear gradually, not instantly.

Meanwhile, the simple complexity apparent in the recurring motif indicates that complex systems can still exhibit predictable behavior. The philosophy, with appropriate mathematical model approach, chaos can be changed become order.

Lastly, connectivity system depicted from every interconnected segments connected, signifies No There is a truly compartment isolated. This is reflect principle conservation population (N = S + I + Q + R) and describes that health is not quite enough answer collective; each compartment in system each other influence and contribute to balance overall.

Meaning of Mathematics

Each type of flower represents a different variable in the SIQR model, namely S, I, Q, and R. The varying flower sizes depict different parameter values, such as μ , β , α , γ , and p. The sigma symbol (Σ) in the center represents the principle of population conservation, namely S + I + Q + R, while the pi symbol (π) indicates a constant or proportion of the population in the calculation. In a multi-compartment system, large flowers represent the dominant compartment (e.g., S or R when $R_0 < 1$), while small flowers represent minor compartments such as I and Q when the disease is under control. Distribution spatial flower depicts proportion relatively each compartment in condition equilibrium .

The stems and branches that connect the flowers reflect connection between variables in systems, such as β si (S–I interaction), α i (I–Q transition), and γq (Q–R transition). The patterns that appear experience describe nonlinear properties of the system, especially in the β si term.

Meaning .

The variety of flora depicts biodiversity population. Big flowers represent individual susceptible adults, buds symbolizes baby new born in to compartment S or direct get vaccine (enter R), flowers bloom full describe individuals who have recovered, while the flowers wilted show infected individuals. Flower stages of bud until fall become analogy flow SIQR compartment: bud (newborn), bloom (phase healthy), wilted (infected) or quarantined), and died (death) with rate μ). Differences in flower conditions also illustrate variations in immune status within the population. Perfect flowers represent individuals with full immunity from the vaccine (proportion p), while imperfect flowers represent susceptible individuals (proportion 1−p).

Closely spaced flowers reflect disease transmission through direct contact, as assumed by the diphtheria model. Flower density indicates density influential population to rate β transmission. Meanwhile that, the flower that grows separated describe individual in quarantine (Q) which is not do contact with population vulnerable, so that lower β si interaction.

Philosophical Meaning

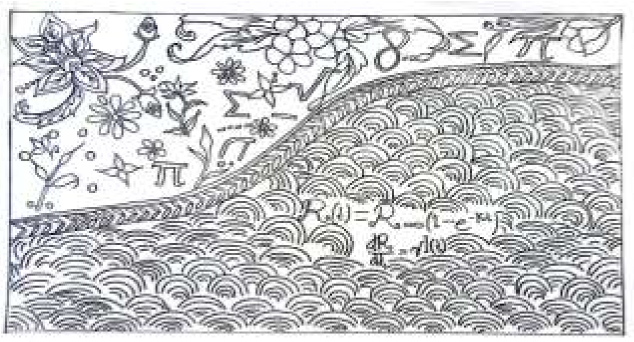
The diversity of flowers reflects the heterogeneous yet interconnected nature of human life. The philosophy is that disease doesn't choose who it attacks, but with the right vaccination, everyone can be protected.

A beautifully blooming flower represents a healthy population ($R_0 < 1$), and optimal conditions are achieved when $p > 0.884$ and $\alpha > 0.049$. This reflects that with the right "nutrition," namely good health interventions and policies, the population will thrive.

The ecosystem balance indicated by the composition of the flora represents a disease-free equilibrium point (E_0). If a "pest" appears, the system is disrupted and becomes endemic (E_1), but balance can be restored through interventions such as vaccination. The continued emergence of buds represents the regeneration of a new population, where vaccination (p) immediately enters the immune compartment (R). Philosophically, the new generation is the hope for a disease-free population.

No flower stands alone; they are all interconnected, reflecting the conservation of population proportions (s + i + q + r = 1). This means that collective health depends on the contributions of all compartments.

Ultimately, the beauty of harmoniously arranged flora represents an asymptotically stable system. If $R_0 > 1$ without intervention, the system will descend into chaos. Philosophy suggests that appropriate science and policy can create order from potential chaos.



Conclusion

Study This succeed developed a mathematical model of SIQR (Susceptible–Infected–Quarantined–Recovered) for analyze dynamics distribution disease diphtheria with considering two forms intervention main, namely vaccination (p) and quarantine (α). This model formulated in form system equality differential nonlinearity that describes change amount individual in every compartment to time. Based on results analysis theoretically, two points are obtained equilibrium main, namely equilibrium free disease (E_0) and equilibrium endemic (E_1). Analysis stability show that system will reach stability asymptotic local on E_0 when $R_0 < 1$, which means disease can controlled and disappeared from population, whereas system will stable in E_1 when $R_0 > 1$, which describes condition disease endemic.

Number value reproduction base (R_0) is proven become a key parameter in determine behavior system. When R_0 decreases below one, population infected (I) and quarantined (Q) will decrease in a way significant until approach zero, whereas population recovered (R) increased going to mark equilibrium R ∞. Conversely, if R_0 is greater big from one, disease still stay inside population in condition endemic. This is show that improvement proportion vaccination (p) and effectiveness quarantine (α) in direct lower R_0 value and increase opportunity system For reach condition free disease.

Simulation results numeric support analysis In the condition $R_0 < 1$, the curve population show decline fast amount individual infected and increased population healed in a way exponential going to point equilibrium. Meanwhile, in the condition $R_0 > 1$, the system show behavior oscillation muffled going to equilibrium endemic, which means although distribution disease can controlled, disease No fully is lost without existence improvement intervention. With Thus, it is necessary effort consistent in guard level vaccination minimum $p > 0.884$ and effectiveness minimum quarantine $\alpha > 0.049$ so that the disease can controlled in a way sustainable.

Furthermore, this research presents a visual and philosophical approach through the integration of mathematical models into scientific batik motifs. The concentric wave, zigzag, and floral patterns in batik not only serve as aesthetic representations but also have scientific and biological meanings. The concentric pattern depicts the cyclical spread of disease, the overlapping waves symbolize the inter-compartmental interaction (SIQR), and the layered structure indicates the iteration of time in a dynamic system. The zigzag pattern represents the relationship between interrelated differential equations and illustrates the cycle of transmission and intervention. The floral motif, with its various sizes and shapes of flowers, symbolizes the proportion of the population and the health status of individuals within the system, ranging from vulnerable to recovered.

Philosophically, this batik motif reflects the balance between order and instability in living systems. A regular pattern indicates system stability when $R_0 < 1$, while a wide and irregular pattern depicts an uncontrolled outbreak when $R_0 > 1$. The central philosophy is that balanced population health can be achieved through knowledge, intervention, and collaboration, with each compartment playing a vital role in maintaining the sustainability of the social and biological ecosystem.

With Thus, research This give contribution theoretical, empirical, and cultural in a way simultaneously. In a way theoretically, the SIQR model was developed strengthen understanding to dynamics distribution diphtheria and effectiveness intervention. Empirically, simulations show that increased vaccination and quarantine effectively reduce the R_0 value to below one. Culturally, the integration of epidemiological models in the form of mathematical batik presents a new way to communicate science through cultural media. These results collectively confirm that science and art can synergize in efforts to achieve a healthy, harmonious, and informed society.

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