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The Spread Pattern of COVID-19 Disease Using Stochastic Differential Equation Susceptible Infected Susceptible Model

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Abstract: Epidemic is the major transmission of an infectious disease that spreads quickly in a large area and causes many victims. Epidemic model is one of the tools that can be used to study the pattern of disease outspread. The SIS model describes the transmission of disease from an individual who is susceptible then infected, directly or indirectly, and becomes an infected individual. Individuals who have been infected can recover but remain susceptible to re-infection because they do not have permanent immunity. COVID-19 is an infectious disease caused by the SARS-CoV-2 virus, an individual can be infected with this disease by breathing air that containing the virus if they are standing close proximity with individuals who are already infected with COVID-19. The purpose of this study was to see the pattern of transmission of infectious diseases using SDE SIS through model simulation and provide interpretation. In this study, rate of contact affected the duration of infectious diseases outspread. The study was conducted by applying a model simulation, given the parameter values of γ (percentage of the population who recovered each period) and β (probability of infection contact between infected individuals and susceptible individuals). The simulation results for the values of $\beta = 0{,}5$ and $\gamma = 0{,}267$ show that the graph of the number of infected individuals increases sharply and tends to be constant at $t = 28$. The number of infected does not decrease again due to the characteristics of the SIS epidemis model, where individuals who recover do not have permanent immunity to the disease, so that the individual becomes susceptible to the disease again.

Keywords: COVID-19, epidemic, *SIS*, stochastic differential equation, simulation.

Introduction

Epidemic is the spread of an infectious disease that spreads quickly in a large area and causes many victims. Infectious diseases can be transmitted from an infected individual to a healthy or susceptible individual through direct or indirect contact. Infectious diseases were caused by microorganisms. COVID-19 disease is an infectious disease caused by the SARS-CoV-2 virus, healthy individuals can become infected with this disease when breathing air containing the virus if they are in close proximity to individuals who are already infected with COVID-19. This COVID-19 disease causes an increase in the death toll and also affects the economic downturn, so a tool is needed to see the pattern of the spread of this COVID-19 disease.

Epidemic model is one of the tools that can be used to study the pattern of disease spread. According to Nasell (2002), in a mathematical model to describe the pattern of spread of infectious diseases, a population is divided into several subpopulations, including a subpopulation of individuals who are susceptible to a disease or what is called Susceptible (S), a subpopulation of individuals who have been infected with a disease but have not been able to transmit it to other individuals or it can be said that the subpopulation is in a latent period called Exposed (E), a subpopulation of individuals infected with a disease or called Infected (I) and a subpopulation of individuals who recovered from a disease or called Removed (R). The epidemic models include

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SI, *SIS*, *SIR* and *SEIR*. These models have different forms of distribution and characteristics. The model of *SIS* describes the spread of the disease from an individual who is susceptible to the disease then becomes infected directly or indirectly and becomes an infected individual. Individuals who have been infected can recover but remain susceptible to re-infection because they do not have permanent immunity.

The model of *SIS* can be viewed in a deterministic or probabilistic manner. The distribution pattern is probabilistically divided into three, namely Discrete Time Markov Chain (*DTMC*), Continuous Time Markov Chain (*CTMC*), and Stochastic Differential Equation (*SDE*). The model of *SDE* is a disease spread over a continuous time interval. Allen (2007) discusses the stochastic model of the epidemic *SIS* model by building on the approach of *SDE*.

The spread of disease here is defined as a random event that depends on the time variable or is called a stochastic process. Changes in the number of infected individuals are considered as a stochastic process in continuous time intervals. Based on the characteristics of the spread of the disease in accordance with the description, this study will use the model *stochastic differential equation susceptible infected susceptible* (*SDE SIS*).

Materials and Methods

The Epidemic SIS Model

Model epidemic Susceptible Infected Susceptible (SIS) according to Tassier (2005) is a model of an epidemic that has characteristics that each individual *susceptible* or not in direct contact with people *infected* will be infected with the disease. Then, using medical treatment or through natural processes, individuals who have been infected can recover, and again become susceptible to disease again, depending on the body's power. There are two population groups in this epidemic *SIS* model, they are *susceptible* (S) and *infected* (I).

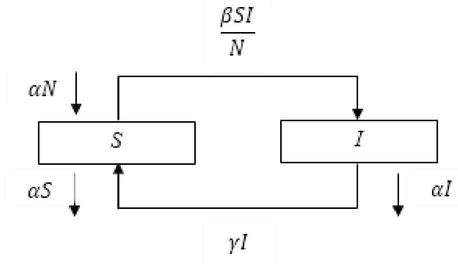


Figure 1. Diagram of the *SIS* model.

Stochastic process

According to Praptono (1986) the stochastic process is a set of random variables which is a function of time or is often called a *random process*. According to Allen (2003), a stochastic process is a collection of random variables $\{X_t, t\, T\}$, with T is the time set of the experiments performed. If $T = (0,1,2, \dots)$, then this process is called a stochastic process with discrete time, whereas if $T = [0, \infty)$, then this process is called a continuous time stochastic process. Index t as time and X_t as position of stochastic process at t time. Changes in the number of infected individuals are random events that appear in the model of *SIS*, so the random variable studied in this study is $I(t)$.

The Wiener Process

In 1827, when the English botanist Robert Brown observed the movement of pollen trapped in a drop of water and moving continuously in haphazard zigzagging trajectories. In 1923, Norbert Wiener presented a mathematical model for the movement that corresponds to a stochastic process. This mathematical model is known as the Wiener process which is denoted by $W(t)$. The following is the definition and theorem of the Wiener process according to Taylor and Karlin (1998).

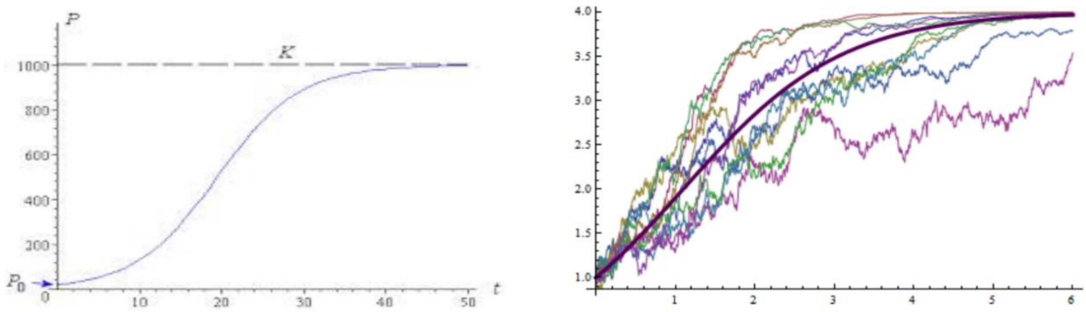
Definition 1. *A stochastic process* $\{W(t), t \geq 0\}$ *is called a Wiener process if it fulfills the assumption*

1. *Value* $W(0) = 0$,
2. *For all* $0 \leq t_1 < \cdots < t_n$, *increase* $W(t_n) - W(t_{n-1}), \dots, W(t_2) - W(t_1)$ *are mutually independent, meaning that the value of increase does not depend on past circumstances,*
3. *For* $0 \leq s < t$, *the increase in* $W(t) - W(s)$ *is distributed normal* $N(0, ts)$.

Theorem 1. *Variable $dW(t)$ with $W(t)$ is a Wiener process, normally distributed with zero mean and variance dt.*

The Epidemic Model of Stochastic Differential Equation

The model of *SIS* can be viewed in a deterministic or probabilistic manner. The distribution pattern is probabilistically divided into three, namely Discrete Time Markov Chain (DTMC), Continuous Time Markov Chain (CTMC), and Stochastic Differential Equation (SDE). The model of SDE is a disease spread over a continuous time interval. Allen (2007) discusses the stochastic model of the epidemic SIS model by building on the approach of SDE.According to Oksendal (2003), the Stochastic Differential Equation (SDE) is a differential equation in which one or more parameter values are stochastic processes and produce a stochastic solution in the form of a model. This SDE can be built from the existing deterministic equations. Usually in the stochastic differential equation there are elements that affect the results of the phenomenon under study. This element is often referred to as white noise. In addition, SDE also contains the Wiener process or Brownian motion. SDE can be solved by the Ito integral method. According to Higham (2001), SDE models have a very important role in various industrial fields, such as economics, finance, biology, chemistry, epidemiology, and also microelectronics.



(a) (b)

Figure 2. (a) Graph of the Deterministic Differential Equation, (b) Graph of the *Stochastic Differential Equation*

COVID-19 Disease

COVID-19 disease is an infectious disease caused by the SARS-CoV-2 virus, healthy individuals can become infected with this disease when breathing air containing the virus if they are in close proximity to individuals who are already infected with COVID-19, besides that individuals can also become infected by touching their eyes, nose, or mouth after touch surfaces that are contaminated with the virus. Individuals who have been infected will experience mild to moderate symptoms and recover without special treatment, but there are also symptoms that cause severe illness and require special assistance. Individuals who recover do not have permanent immunity to this virus, so they have the possibility of being re-infected and/or reverting to being susceptible individuals.

Methodology

Methods used in this study are literature review, analysis, and preparation of simulations. The steps that will be carried out in this study, namely collecting data related to the spread of the disease with assumptions to support the epidemic model of SDE SIS. The next step is simulating case examples using software Matlab to provide an overview of the pattern of the spread of the disease, and finally draw conclusions.

The assumptions used in this study are as follows (a) the population is closed and the number of individuals in the population is constant, (b) the population is homogeneously mixed, (c) the birth rate is the same as the death rate, (d) individuals born are individuals who healthy but susceptible to disease, (e) recovered individuals are considered to have no permanent immunity, so they can be re-infected, and (f) there is only one disease that is spreading in the population.

Results and Discussion

List of variables and parameters in the mathematical model is presented in Table 1 and Table 2 below.

Table 1. List of Variables.

No	Variable	Description
1	N	Total human population
2	S	Number of susceptible individuals (*susceptible*)
3	I	Number of infected individuals (*infected)*

Table 2. List of Parameters.

No	Parameter	Description
1	α	Birth rate in the population is assumed to be equal to death rate
2	β	Probability of infection contact (contagion) between infected individuals and susceptible individuals
3	γ	Recovery rate of each infected individual

The values of the parameters above are positive (> 0).

The epidemic model of *SDE SIS* applied to the spread of COVID-19 disease referring to Otunuga (2021). The data used is COVID-19 data on United State which was taken at March 1st until November 30th, 2020 with a contact rate value of = 0,5 per day, recovery rate = 0,267 per day, birth rate equal to death rate = 0,0002 per day.

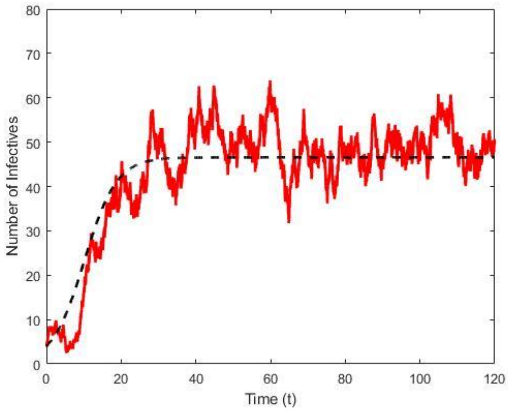


Figure 3. Number of infected individuals at an interval of $0 \leq t \leq 120$

Figure 3, the black dotted line shows the number of infected individuals and only considers the deterministic part, namely the value of indicates the number of infected $\mu(I)$, while the red line individuals. infected by considering the deterministic part and the stochastic part, namely the values of $\mu(I)$ and $\sigma(I)$. Figure 3. shows the number of infected individuals when the parameter value of contact rate = 0.5, cure rate = 0.267, and α which is the birth rate and death rate which is assumed to be the same at 0.0002.

From these two lines, it can be seen that with increasing time, the number of infected individuals increased sharply which then increased slowly and tended to be constant after $t = 28$. Furthermore, the number of infected individuals did not decrease due to the characteristics of the epidemic model *SIS* itself, namely individual I that recovered did not have permanent immunity to the disease so that it returned to being an individual S.

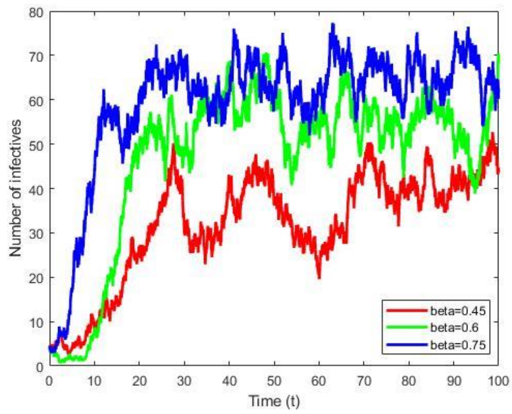


Figure 4. Number of infected individuals with $\beta > \gamma$

Figure 4. shows the number of infected individuals when the parameter value of contact rate more than recovery rate. From the three lines, it can be seen that with the parameter value of contact rate more than recovery rate can make

faster increase in the spread of the disease and the number of infected increased faster too.

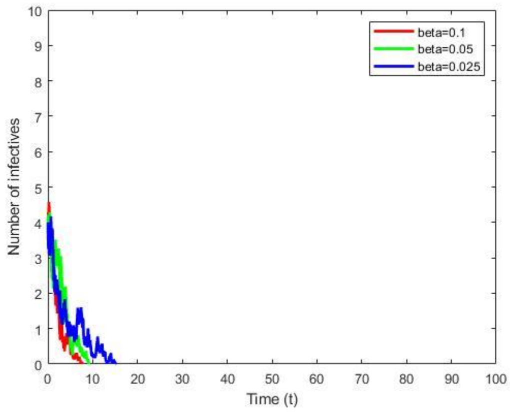


Figure 5. Number of infected individuals with $\beta < \gamma$

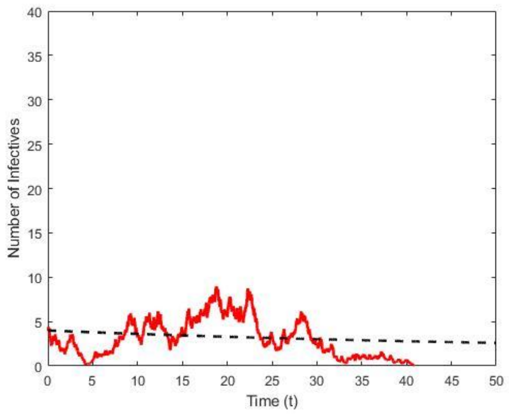


Figure 6. Number of infected individuals with $\beta = \gamma$

Figure 5, shows the number of infected individuals when the parameter value of contact rate less than recovery rate and Figure 6 shows the number of infected individuals when the parameter value of contact rate equal to recovery rate From Figure 5 and 6, it can be seen that the number of infected individuals did not decrease and the transmission of diseases no longer spread.

Conclusions

Based on the discussion, the simulation results for the values of $\beta = 0{,}5$ and $\gamma = 0{,}267$ show that the graph of the number of infected individuals increases sharply and tends to be constant at $t = 28$. The number of infected does not decrease again due to the characteristics of the *SIS* epidemis model, where individuals who recover do not have permanent immunity to the disease, so that the individual becomes susceptible to the disease again. The results of the application of the COVID-19 disease were obtained in the form of a graph showing the pattern of the spread of the COVID-19 disease in the number of infected individuals which increased with time.

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