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ANALYSIS OF STABILITY OF HIV/AIDS EPIDEMIC IN YOGYAKARTA WITH AGE GROUP AND POPULATION DENSITY

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Abstract

HIV/AIDS is a disease caused by a virus that attacks the immune system, causing the body's immunity to decrease. In this study, the SI (Susceptible-Infected) mathematical model will be studied to analyze the stability of the HIV/AIDS epidemic model, especially in the Special Province of Yogyakarta (DIY) based on age group and population density. The age group is divided into two subpopulations, namely children and adults. The analysis carried out is to determine the local and global stability of the equilibrium point of the disease-free and disease-infected model. The analysis uses the characteristic equation of the Jacobi matrix and the Lyapunov-La Salle invariance principle or uses the conditions of the threshold value of susceptible reproduction ratio (R_1), infected reproduction ratio (R_0), and infectious contact rate. (R_2). For cases of HIV/AIDS data in the Province of the Special Region of Yogyakarta (DIY) with an initial population of 2016, obtained $R_0 = 0.027$, $R_1 = 114.25$, $R_2 = 0.93$. For cases of HIV/AIDS data in the Province of the Special Region of Yogyakarta (DIY) with an initial population of 2016, obtained $E_1 = (924.280, 179.402, 0, 0)$. The disease-free equilibrium point is globally asymptotically stable, meaning that if the parameter values do not change then there are no infected individuals and the susceptible subpopulation of children and adults to a constant positive value.

Keywords: Equilibrium Point; HIV/AIDS; Local and Global Stability; Reproduction Ratio; SI Mathematical Model.

1. INTRODUCTION

HIV or Human Immunodefiency Virus is a disease caused by a virus that attacks the immune system, causing the body's immunity to decrease. Someone who is positively infected with HIV within five to ten years later that person will suffer from AIDS (Acquired Immunodeficiency Syndrome). HIV endemic is still a serious problem in the world of health. Until now there is no medicine to treat HIV and AIDS. However, there are drugs to slow the progression of the disease, and can increase the life expectancy of sufferers. The Special Region of Yogyakarta (DIY) is a province in Indonesia that has potential in the fields of tourism, education and culture. DIY is known to have a large number of HIV/AIDS cases. In 2019, HIV/AIDS cases in DIY have shown red, which means it is very dangerous and must be stopped immediately. Therefore, the role of all parties is needed, starting from the community, health practitioners and the government so that the prevention and control of the spread of HIV/AIDS can be further suppressed.

Mathematical models can play an important role in preventing high cases of HIV/AIDS in the world. Based on the transmission mechanism of the HIV model, mathematical models can help the medical and scientific communities understand and anticipate its spread in different populations and evaluate the potential effectiveness of different approaches to keep epidemics under control Hyman and Stanley (1988).

The HIV/AIDS epidemic model used in this study refers to Marsudi's research (2014), namely the dynamics of HIV/AIDS disease with an age structure where subpopulations are modeled using the SI (Susceptible, Infected) epidemic model which corresponds to sexually transmitted diseases without recovery. The model is described through two age subpopulations, namely children (0-14 years) and adults (15 years and over). Epidemiologically, the subpopulation of children consists of the Susceptible class of children (S_a) and Infected children (I_a). The adult sub-population consists of the adult Susceptible class (S_d) and adult infected (I_d). If the population continues to increase and while the demographic or environmental conditions have a limited number of resources, population density will occur due to internal competition and affect survival and can even cause the death of individuals in the population. We examine the HIV/AIDS epidemic model with the influence of age groups and population density using HIV/AIDS data in the Special Province of Yogyakarta.

2. METHOD

The writing and preparation of this study used the literature study method, namely several sources of literature that discuss mathematical modeling related to HIV/AIDS. The research was conducted using secondary data in the form of demographic data, empirical data on HIV/AIDS in DIY. Demographic data were obtained from the DIY Central Bureau of Statistics, while empirical data on HIV/AIDS were obtained from the DIY Health Office and from relevant literature. The stages of the research carried out were estimating model parameters, formulating a deterministic model (SI model) with a system of nonlinear differential equations, determining the equilibrium point of the model obtained, conducting stability analysis of local and global equilibrium points and implementing the model on case data. HIV/AIDS in the Province of the Special Region of Yogyakarta (DIY) as well as conducting numerical simulations with the Matlab program. The HIV/AIDS epidemic model is described using a compartmental model which schematically transitions between the two subpopulations can be presented in the diagram in Figure 1 (López, et. al., 2007).

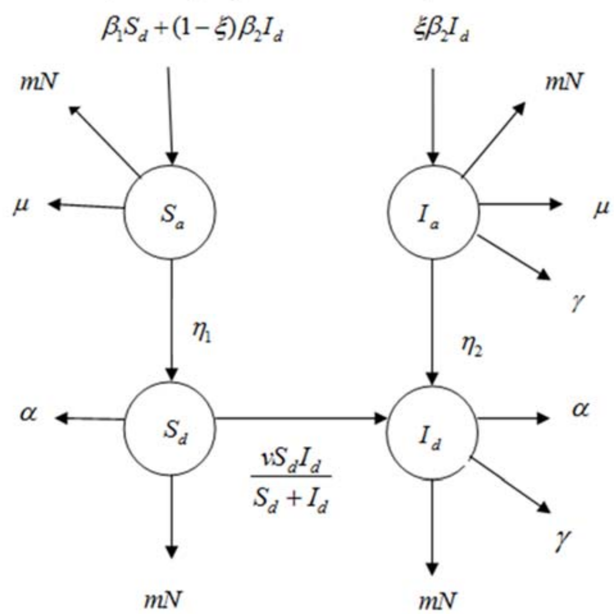


Figure 1. Diagram of the HIV/AIDS Model with Two Age Groups

Based on Figure 1, the model can be described with four nonlinear differential equations (Marsudi, 2014)

$$\frac{dS_a}{dt} = \beta_1 S_d + (1-\xi)\beta_2 I_d - \mu S_a - mS_a N$$
$$\frac{dS_d}{dt} = \eta_1 S_a - \frac{vS_d I_d}{S_d + I_d} - \alpha S_d - mS_d N$$
$$\frac{dI_a}{dt} = \xi\beta_2 I_d - \eta_2 I_a - \mu I_a - \gamma I_a - mI_a N$$
$$\frac{dI_d}{dt} = \eta_2 I_a + \frac{vS_d I_d}{S_d + I_d} - \alpha I_d - \gamma I_d - mI_d N$$

where $\beta_1(\beta_2)$ is the birth rate per capita of the average susceptible (infected) adult, $\eta_1(\eta_2)$ is the per capita maturation rate of susceptible (infected) children, $\mu(\alpha)$ is the natural death rate of the subpopulation of children (adults), γ is the death rate due to HIV per capita and is the contact rate per capita between susceptible individuals and adult infected individuals. We assume that all parameters of the system are positive. We assume the population density coefficient is positive and $\xi(0 \leq \xi \leq 1)$ explaining that the portion of babies born to infected adult groups is infected. We assume no migration, interpret maturity as emigration from the childhood level and immigration into the adult level, individuals are not immune, only adults can give birth, the death rate of the suspected child subpopulation is identical to the death rate of the infected child subpopulation, and there is no infection due to sexual contact in a subpopulation of children.

3. RESULTS AND DISCUSSION

Based on vertical data from the Central Statistics Agency (Fauk, et.al., 2021), the Special Region of Yogyakarta (DIY) is a province in Indonesia with a population of 3,720,912 people with a population density 1,168 soul/km^2. The composition of the DIY Province population by age group shows that the population is young or children (0-14 years) of 21; 77% which is 810,186 people and the adult group (over 15 years) is 78.23% or 2,565,765 people. The composition of the population by sex shows that there were 400,557 boys, 1,439,394 girls, 409,485 adult men and 1,471,476 adult women.

Based on data from the DIY HIV/AIDS Commission (Marsudi, 2014), HIV/AIDS cases in DIY from 1993 to March 2016 totaled 4,648 people consisting of 3,089 men, 1,481 women, and 78 unknown. By age group there were 162 children, 4,147 adults, 338 unknown. Throughout 2015, there were 405 cases with details of 313 HIV cases and 92 AIDS cases. Each individual can be infected with HIV with an average duration of 8.6 – 19 years (Marsudi, 2014). Then, based on data from the DIY Province Statistics Agency, life expectancy in 2016 was 75 years. The child mortality rate in 2007 – 2010 was 16 (out of 1000) and in 2010 – 2012 it was 25 (out of 1000). The average number of children born to adult women is around 2.02%. The maturation rate for children is assumed to be the same as for adults, which is around 16 years. The estimated population of Indonesia (in thousands) for 8 years (2004 – 2011) grouped at one year intervals is presented in Table 1 and the graph is shown in Figure 2.

Table 1. Estimated Number of Population (in thousands)

Year	2012	2013	2014	2015	2016	2017	2018	2019
Total population	3552	3594	3637	3679	3720	3762	3802	3842

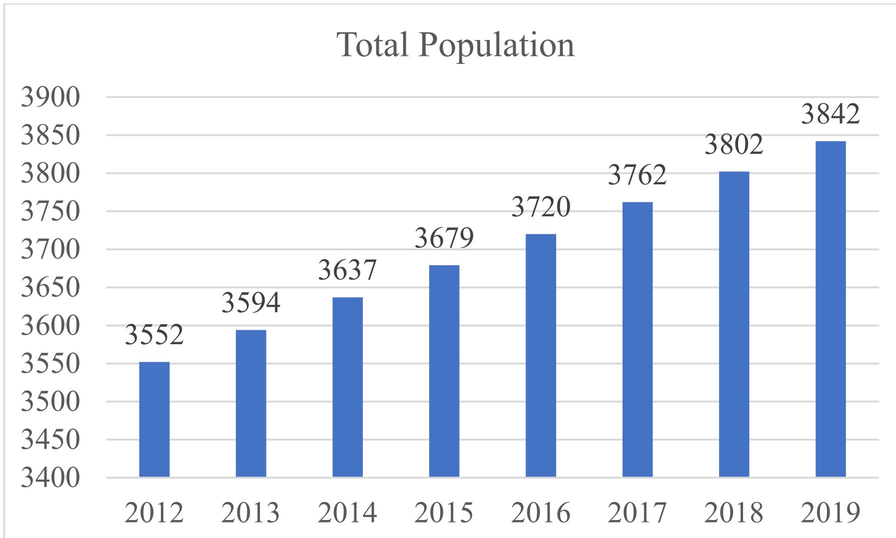


Figure 2. Indonesian Population Data for Eight Years

The graph of population to year in Figure 2 appears to be in a logistic form with a differential equation $\frac{dN}{dt} = aN(K-N)$ where N is the total population of Indonesia at that time t, K is the final value (carrying capacity) and r is the kinetic parameter (intrinsic rise rate). Score K and a determined by the approximation of the differential equation, namely by the differential equation $\frac{\Delta N}{\Delta t} = aN(K-N)$.

3.1. Estimation of Model Parameter Values

The estimation of the model parameter values can be determined from the DIY Province data which has been described previously (López, et. al., 2007), including:

a. Per capita birth rate of the average susceptible adult per year (β_1), i.e. the total susceptible female population multiplied by the birth rate which is then divided by the total female population

$$\beta_1 = \frac{(1469995)(2.02)}{1471476}.$$

b. Per capita birth rate from the average adult infected per year (β_2), namely the total infected female population multiplied by the birth rate which is then divided by the total female population

$$\beta_2 = \frac{(1{,}481)(2.02)}{1471476}.$$

c. Child mortality rate per capita per year (μ), i.e. the average child mortality rate (JDR) of two different years

$$\mu = \frac{JDR(2007-2010) + JDR(2010-2012)}{2} = \frac{{}^{16}/_{1000} + {}^{25}/_{1000}}{2} = 0.0205.$$

d. Death rate per capita per year (α), defined as the inverse of the mean life expectancy at birth (ALE)

$$\alpha = \frac{1}{\text{ALE}} = \frac{1}{75} = 0.0133.$$

e. HIV/AIDS death rate per capita per year (γ), define the average remaining life of adult infected individuals equal to the inverse of the mean infection period (MIP). We assume MIP is equal to 17 years (Marsudi, 2014)

$$\gamma = \frac{1}{\text{MIP}} = \frac{1}{17} = 0.05882.$$

f. Contact rate (v). Because the total suspect population is close to the total population, the contact rate can be calculated from the total newly infected adults in one year (data taken from 2015) which is then divided by the total infected population.

$$v = \frac{313}{4648} = 0.06734$$

g. Maturation rate of susceptible children (η_1) assumed to be the same as infected children (η_2)

$$\eta_1 = \eta_2 = \frac{1}{16} = 0.0625$$

h. Internal Competition (m) calculated by formula $m = \frac{\beta_1 - \alpha}{K}$, where K is Indonesia's carrying capacity due to population density caused by internal competition. Carrying capacity due to internal competition is estimated using Indonesian data for eight years (2012 – 2019). Value K and a can be derived by checking the maximum value of the function

$$y(\text{N}) = \frac{dN}{dt} = aN(K - N)$$

From $\frac{dy}{dN} = 0$, we get $a(K - 2N) = 0$ and $N_{maks} = \frac{K}{2}$. Table 2 is an estimate of $y(\text{A}_{maks})$.

Table 2. Estimation of $y(\text{A}_{maks})$

$\Delta y_i = A_{i+1} - A_i$	Nilai maksimum y
42.4	$y(\text{A}_{maks}) = y\left(\frac{K}{2}\right)$
42.3	
42	
41.7	
41.3	
40.1	
40	

To determine the relationship between a and K, then enter the value N = 3573.5 which is the average where the maximum value is found i.e. the population in 2012 and 2013.

$$a(3573.5)(K - 3573.5) = 43 \Rightarrow a = \frac{0.012}{K - 3573.5}$$

We have known before $y(\text{A}_{maks}) = y\left(\frac{K}{2}\right)$. Therefore, to get value K is

$$aN(K - N) = 43$$

$$\left(\frac{K}{2}\right)^2 = \frac{43(K-3573.5)}{0.012}$$
$$K^2 = \frac{4(43)(K-3573.5)}{0.012}$$

It is known from the equation above $K_1 = 7948$ and $K_2 = 6385$. So that we get $K = 7166.5$ which is the average of K_1 and K_2. Because the data is estimated in thousands, the estimated carrying capacity, then the population density (K) is 7,166,500, so

$$m = \frac{\beta_1 - \alpha 4(43)}{K} = \frac{2.018 - 0.0133}{7166500}.$$

Thus, the estimated parameter values for the HIV/AIDS epidemic model for Indonesia are presented in Table 3.

Table 3. Parameter Value Estimation

No.	Parameter	Value (per Year)
1.	β_1	2.018
2.	β_2	0.002
3.	μ	0.0205
4.	α	0.0133
5.	γ	0.05882
6.	v	0.06734
7.	η_1	0.0625
8.	η_2	0.0625
9.	m	2.797×10^7
10.	ξ	0.5

3.2. Reproductive Ratio Threshold Value

By using the previously described formulas and the estimated values of the model parameters in Table 3, the following is the threshold value of the susceptible reproduction ratio (R_1), infected reproduction ratio (R_0), and infectious contact rate (R_2).

a. Susceptible Reproductive Ratio Threshold Value

Reproduction ratio threshold value is susceptible (R_1) is the average number of susceptible babies born to suspect individuals during their lifetime. We use the model parameter values in Table 3,

$$R_1 = \left(\frac{\beta_1}{\alpha}\right)\left(\frac{\eta_1}{\eta_1 + \mu}\right) = 114.25$$

So, the value of the susceptible reproduction ratio is greater than one.

b. Infected Reproductive Ratio Threshold Value

The infected reproduction ratio threshold value (R_0) is the average number of new susceptibles produced by susceptible individuals. We use the model parameter values in Table 3.

$$R_0 = \left(\frac{\xi\beta_2}{\alpha + \gamma}\right)\left(\frac{\eta_2}{\eta_2 + \mu + \gamma}\right) = 0.027$$

So, the value of the susceptible reproduction ratio is less than one.

c. Infectious Contact Rate Threshold Value

Infectious contact rate threshold value (R_2) is the average number of contacts of adult infected individuals during their lifetime. We use the model parameter values in Table 3, $R_1 = \frac{v}{\alpha+\gamma} = 0.93$, So, the threshold value of the infectious contact rate is less than one.

3.3. Model Stability Analysis

The system has a disease free equilibrium point $E_1^* = (S_a^*, S_d^*, 0, 0)$, where $S_a^* = \frac{(\alpha + mN_1^*)N_1^*}{\alpha + \eta_1 + mN_1^*}$ and $S_d^* = \frac{\eta_1 N_1^*}{\alpha + \eta_1 + mN_1^*}$, which then looks for the value of N_1^*, that is $N_1^* = \frac{-(\eta_1 + \mu + \alpha) + \sqrt{(\eta_1 + \mu + \alpha)^2 - 4[\alpha(\eta_1 + \mu)(1 - R_1)]}}{2m}$ if $R_1 > 1$.

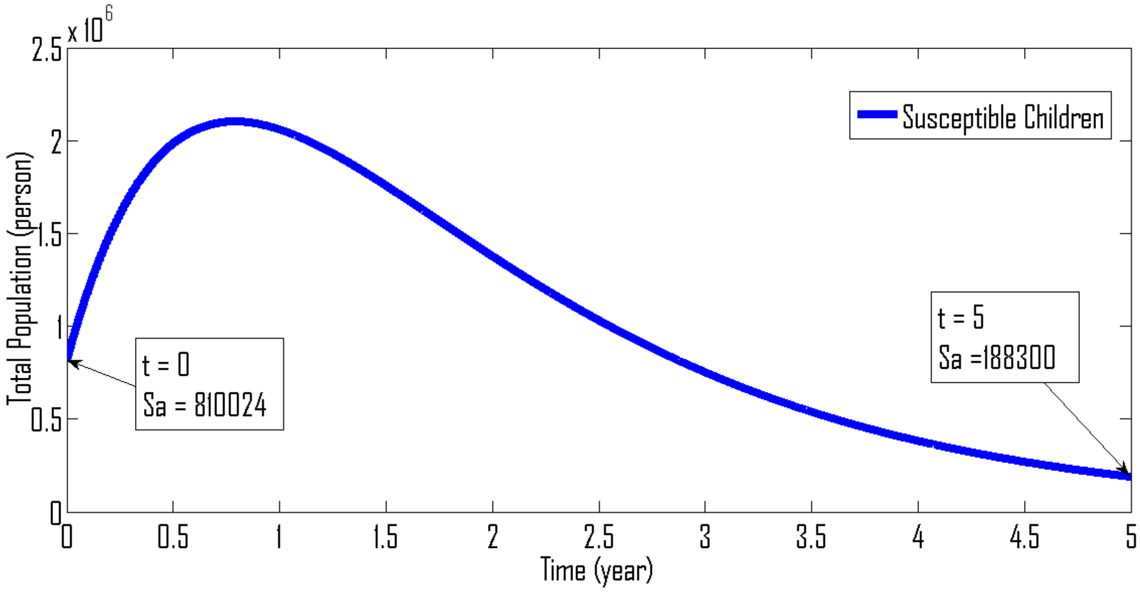
Next, the extinction equilibrium point is susceptible $E_2^* = (0, 0, I_a^*, I_d^*)$, where $I_a^* = \frac{(\alpha + \gamma + mN_2^*)N_2^*}{\alpha + \gamma + \eta_2 + mN_2^*}$ and $I_d^* = \frac{\eta_2 N_2^*}{\alpha + \eta_2 + \gamma + mN_2^*}$, where is the value of N_2^* obtained ie $N_2^* = \frac{-(\eta_2 + \mu + \alpha + 2\gamma) + \sqrt{(\eta_2 + \mu + \alpha + 2\gamma)^2 - 4[(\alpha + \gamma)(\eta_2 + \mu + \gamma)(1 - R_0)]}}{2m}$ if $R_0 > 1$ and $\xi = 1$.

According to research conducted by Lopez, et. al. (2007), disease free equilibrium point $E_1^* = (S_a^*, S_d^*, 0, 0)$ locally asymptotically stable if $R_2 < 1$ and $R_0 + R_2 \le 1$ and global stable if $R_1 > \frac{(\eta + \mu)(v + \alpha)}{\eta_1 + \alpha}$ and $R_2 \le 1 - R_0$. The susceptible extinction equilibrium point E_2^* is locally asymptotically stable if $R_1 < 1$, $\xi = 1$, and $R_0 > \frac{\mu + \eta_2 + \gamma}{\eta_2}$.

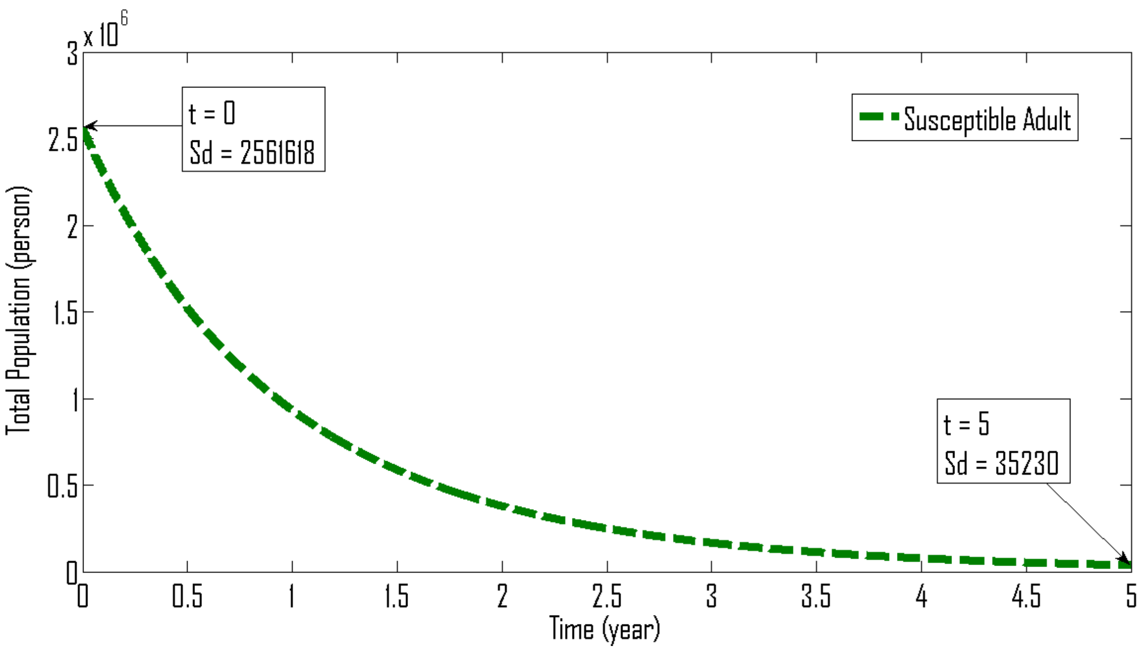
We use the parameter values in Table 3 and therefore $R_1 = 114.25 > 1$, $R_0 = 0.027 < 1$, $R_2 = 0.93 < 0.973 = 1 - R_0$, then the HIV/AIDS model with the influence of age groups and population density in DIY Province only has a disease-free equilibrium point $E_1^* = (924.280, 179.402, 0, 0)$ locally asymptotically stable. This can also be seen from the Jacobi matrix in $E_1^* = (924280, 179.402, 0, 0)$, i.e.

$$J(E_1^*) = \begin{pmatrix} -0.65 & 1.759 & -0.258 & -0.257 \\ 0.012 & -0.372 & -0.05 & -0.117 \\ 0 & 0 & -0.45 & 0.0002 \\ 0 & 0 & 0.0625 & 0.534 \end{pmatrix}.$$

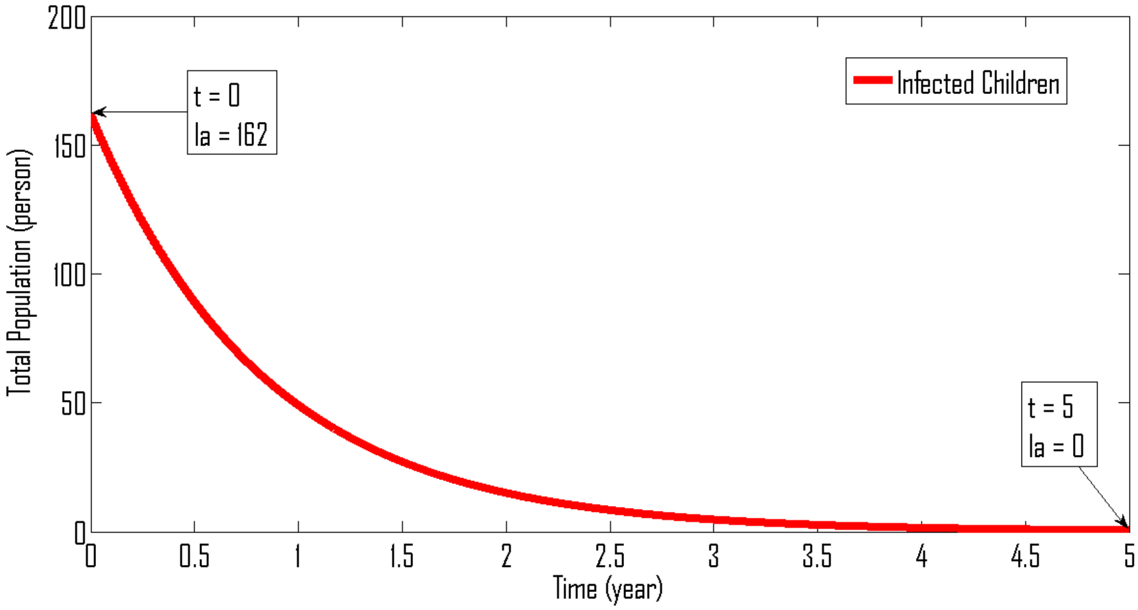
where are all the eigenvalues of $J(E_1^*)$ is negative ie $\lambda_1 = -0.7121$, $\lambda_2 = -0.3099$, $\lambda_3 = -0.7121$ and $\lambda_4 = -0.5341$. Next, because $R_2 + R_0 = 0.957 < 1$ and $R_1 = 114.25 > 2.269 = \frac{\mu + \eta_2 + \gamma}{\eta_2}$, then the equilibrium free disease point $E_1^* = (924.280, 179.402, 0, 0)$ globally asymptotically stable. The global stability of this disease-free equilibrium point can be seen in Figure 3, which is a graph of the numerical solution of the HIV/AIDS epidemic model using the fourth order Runge-Kutta method.



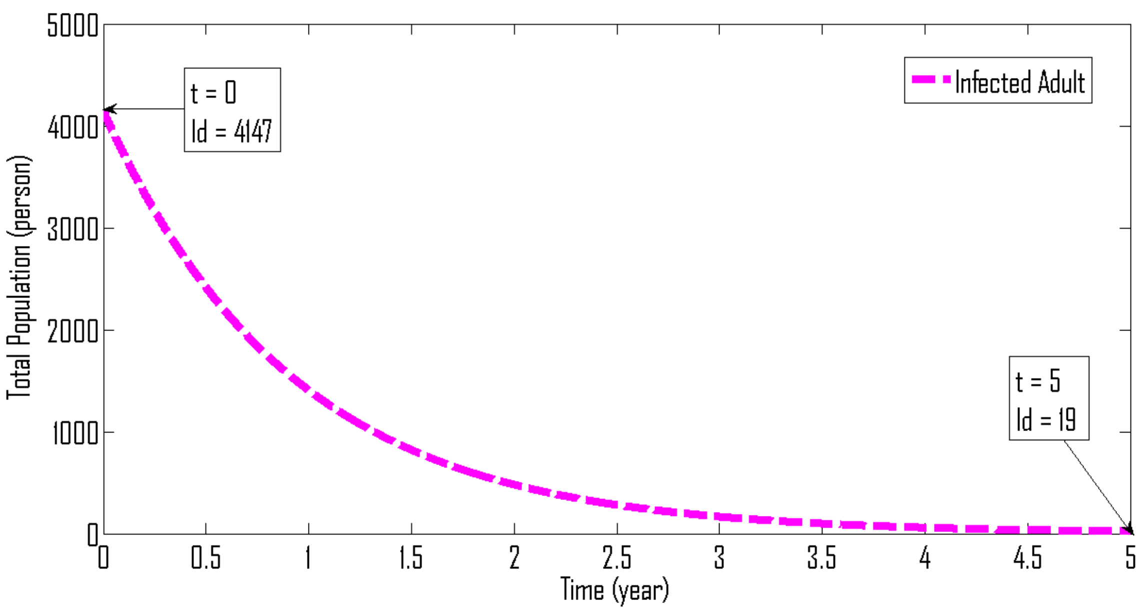
(a)



(b)



(c)



(d)

Figure 3. Portrait of the Phases of the HIV/AIDS Epidemic Model with the Effect of Age Groups and Population Density in the Province of DIY with $S_a(0) = 810024$, $S_d(0) = 2561618$, $I_a(0)$ (0) = 162, and $I_d(0) = 4147$.

Figure 3 (a) shows that the subpopulation is susceptible to children (S_a) tends to increase in the first year, then continues to decrease for the first five years. The subpopulation of susceptible children in 2019 was 810,024 people and in 2024 there were 188,300 people. The subpopulation of susceptible children is converging towards $S_a^* = 924.280$.

Figure 3(b) shows that from the beginning of the year to the first five years the adult susceptible subpopulation (S_d) continues to decrease in number. The susceptible adult subpopulation in 2019 was 2,561,618 people and in 2024 there were 35,230 people. The adult susceptible subpopulation is converging towards $S_d^* = 179402$.

Figure 3(c) shows the subpopulation of infected children (I_a) in the first five years fell. The subpopulation of infected children in 2019 was 162 people and in 2024 there were 0 people. The subpopulation of infected children converges towards $I_a^* = 0$.

Figure 3(d) shows the adult infected subpopulation (I_d) in the first five years it was almost the same as the infected child subpopulation, that is, there was a decrease in the number of adult infected subpopulations. From the beginning of the year to the first five years, the numbers continued to fall and converge $I_d^* = 0$. If the parameters do not change, then there are no infected individuals and the susceptible subpopulation of children and adults towards a constant positive value.

4. CONCLUSION

For cases of HIV/AIDS data in DIY Province with data from the beginning of 2016, it is known that the threshold value of the susceptible reproduction ratio is known R_1 = 114; 25, infected reproduction ratio R_0 = 0; 27 and infection rate R_2 = 0; 93. The HIV/AIDS epidemic model with age group and population density has one disease-free equilibrium point $E_1^* = (924280, 179402, 0, 0)$ which is asymptotically stable globally after approximately six years. If the parameters do not change then there are no infected individuals and the susceptible subpopulation of children and adults will reach a constant positive value after six years.

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