

NUMERICAL ANALYSIS OF A MATHEMATICAL MODEL OF THE DYNAMICS OF CORRUPTION IN INDONESIA USING A FOURTH-ORDER RUNGE-KUTTA METHOD MODIFIED BY THE GEOMETRIC MEAN

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Abstract

Corruption is one of the issues that negatively impacts various aspects of life in Indonesia. This study aims to develop a mathematical model of the dynamics of corruption in Indonesia in the form of a system of differential equations and to analyze its numerical solutions using the fourth-order Runge-Kutta method with a geometric mean modification. The model describes changes in the number of individuals in each compartment over time based on the interactions occurring within the system. The difficulty in obtaining analytical solutions led to the use of a numerical approach employing the fourth-order Runge-Kutta method with a geometric mean modification. The method was implemented using the Python programming language. The results of the study show that the developed model is capable of representing the dynamics of corruption behavior, and numerical simulations using the fourth-order Runge-Kutta method with a geometric mean modification can provide an overview of the system's behavior over time.

Keywords: Corruption, Mathematical Model, System of Differential Equations, Fourth-Order Runge-Kutta Method, Geometric Mean, Numerical Simulation.

1. INTRODUCTION

Indonesia, a country rich in natural resources and with great development potential, still faces serious challenges in the form of criminal acts of corruption (Gurning, 2024). Corruption poses a real threat to sustainable development, economic equity, and the strengthening of state institutions. In recent decades, the problem of corruption in Indonesia has reached alarming levels and has caused widespread harm to society (Saputra, 2023).

The importance of combating corruption is not only related to moral and ethical considerations, but also stems directly from the fact that corruption has impacted economic development, investment, and social stability (Rachmawati, 2022). The international survey and index organization Transparency International indicates that Indonesia still has significant levels of corruption, the effects of which are felt across various sectors, including public services, public financial management, and investment (Trans, 2021).

One of the serious consequences of corruption is the erosion of public trust in the government and public institutions (Azmi, 2024). This can hinder public participation in development and increase levels of social inequality. Therefore, serious efforts are needed to eradicate corruption—not only to comply with legal requirements but also as a strategic step toward building a clean, transparent, and accountable nation.

As awareness of the negative impacts of corruption has grown, a number of researchers have proposed various strategies and approaches to combat corruption in Indonesia. The strategies and approaches proposed in these studies cover legal, social, economic, political, and religious aspects. Various dimensions, ranging from regulatory reforms to strengthening law

enforcement agencies and empowering the public, have also been proposed. However, despite the implementation of various strategies and approaches to combat corruption in Indonesia, empirical evidence indicates that corruption levels in the country remain high and are even on the rise. On the one hand, this phenomenon indicates that all the proposals put forward to eradicate corruption in Indonesia have not yet been fully appropriate or effective in their implementation. Therefore, a more comprehensive alternative approach is needed to eradicate corruption in Indonesia (Sung, 2018).

A number of previous studies have examined strategies for combating corruption through various non-mathematical approaches, covering legal, social, economic, and cultural aspects. In the legal sphere, (Wibisono, 2021) highlights the effectiveness of sting operations as a law enforcement strategy implemented by the Corruption Eradication Commission (KPK). This study emphasizes that repressive approaches can serve as a deterrent, but they must be balanced with broader legal reforms to ensure the sustainability of anti-corruption efforts.

Social approaches have also been a focus of research in anti-corruption efforts. Furthermore, (Marzuki, 2018) examines the role of religious institutions and the values of Pancasila in fostering anti-corruption awareness among the public. This study shows that the internalization of ethical and moral values can reduce individuals' tendency to engage in corrupt practices. Additionally, (Sarono, 2018) highlights the application of Islamic law in combating corruption, emphasizing the importance of Sharia-based sanctions and moral education as preventive strategies.

From an economic perspective, (Arianto, 2021) proposes forensic accounting as a strategic tool for detecting and preventing corruption, particularly in bribery cases. This study reveals that the application of forensic accounting can enhance financial transparency and reduce opportunities for corrupt practices in both public and private institutions.

Although these approaches have contributed to understanding and addressing corruption, this research seeks to offer an alternative strategy for combating corruption in Indonesia, namely through a mathematical modeling approach. Mathematical modeling is a field within mathematics that seeks to analyze and translate real-world phenomena (in this case, corrupt behavior in Indonesia) into mathematical equations that describe those phenomena (Niss, 2020). The development of the mathematical model in this study will take into account the key factors in combating corruption: active public participation, regulatory improvements, and the strengthening of law enforcement agencies. By solving the constructed mathematical model, a qualitative analysis of corrupt behavior in Indonesia will be generated, and a holistic strategy for combating corruption will be derived.

Through this research, it is hoped that a numerical approach can be developed to solve the mathematical model that has been constructed, and that its numerical simulation will be analyzed using Python software. The aim is to make it easier to solve the mathematical model of the dynamics of corruption in Indonesia and to reduce the time required compared to an analytical solution.

2. MATERIALS AND METHODS

This study employs a mathematical modeling approach to understand the dynamics of corruption spread in Indonesia and evaluate strategies for combating it. The model developed classifies the population into five categories: the Vulnerable Group (S), the Immune Group (I), the Corrupt Group (C), the Incarcerated Group (J), and the Reformist Group (R) (Aickramaarachchi, 2021). This approach involves formulating a system of differential equations to describe the interactions between groups and analyzing the factors that influence an individual's transition from one group to another (Kumar, 2021).

Since the resulting system of differential equations is difficult to solve analytically, a modified fourth-order Runge–Kutta method was used as a numerical approach to obtain an

approximate solution. This method is an extension of the classical fourth-order Runge–Kutta method, utilizing an averaging modification in the slope approximation process.

The numerical method was implemented using the Python programming language. The simulation results were then presented graphically to illustrate the dynamics of each population compartment over time. Subsequently, the simulation results were analyzed to understand the system's behavior and the influence of initial conditions on the development of corruption in Indonesia.

3. RESULTS AND DISCUSSION

The mathematical model of the dynamics of corrupt behavior in this article is categorized into five groups: the vulnerable, the immune, the corrupt, the convicts, and the reformers, as follows

3.1. Mathematical model

In this article, the total population (N) is further divided into five groups: the Vulnerable Group (S), the Immune Group (I), the Corrupt Group (C), the Incarcerated Group (J), and the Reformist Group (R). The Vulnerable Group (S) consists of individuals who have never been involved in corrupt practices but are susceptible to being drawn into such practices within society. This group is composed of individuals born with strong moral standards. Individuals vulnerable to corruption face two possible outcomes: they may move to the Corruptors Group or join the Immune Group. They will move to the corruptors' group if they are successfully influenced by their interactions with corrupt individuals. Conversely, they will join the immune group if they remain unaffected by their interactions with corrupt individuals. Similarly, if they possess strong faith, they can draw lessons from every instance of corruption, which will strengthen their resolve not to engage in corruption. Meanwhile, the "Immune" group (I) consists of individuals who possess high moral standards and will never engage in corrupt practices, no matter what happens in their surroundings.

The Corruptors Group (C) consists of individuals who are frequently involved in corrupt practices and are capable of influencing vulnerable parties to become corrupt. Corrupt individuals who receive proper guidance through public awareness campaigns leave the corruptors group and join the reformers group. Corrupt individuals who are prosecuted and imprisoned leave the corruptors group and join the imprisoned group.

The Inmate Group (J) consists of individuals imprisoned for a specific period for committing acts of corruption. While in prison, members of this group cannot engage in corrupt practices nor influence others to do so. These individuals will become reformed after receiving proper guidance through public education. Meanwhile, the Reformed Group (R) consists of former inmates who were reformed while serving their prison sentences but remain vulnerable to corruption. On the other hand, the Reformed Group (R) can still interact with the Immune Group (I). If this is the case, it is hoped that the Reformed Group (R) will become immune permanently (Hearne, 2021).

The mathematical model of the dynamics of corruption described above can be illustrated in the diagram shown in Figure 1 below

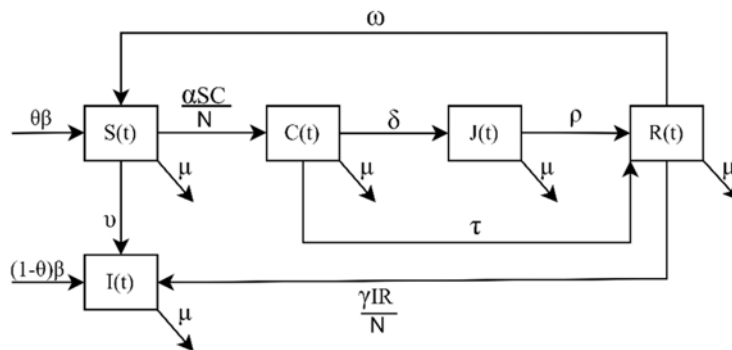


Figure 1. dynamics of corruption

The parameters involved in the diagram above are defined in Table 1 below. These parameters are derived from theoretical assumptions regarding the dynamics of corruption in Indonesia.

Table 1. Parameter Definition

No	Parameter	Description
1	θ	The proportion of individuals who do not have immunity, which is the ability of a person to defend himself from corruption.
2	β	Birth rate of individuals entering the population
3	α	Effectiveness of vulnerable individuals interactions with corrupt individuals
4	γ	Effectiveness of reformed individuals interaction with immune individuals
5	ω	Success rate of reformist individuals turning into corruption-prone individuals
6	v	The degree to which vulnerable individuals become immune to corruption
7	δ	Prosecution and imprisonment rates for corrupt actors
8	τ	Success rate of corrupt individuals becoming reformers due to public enlightenment
9	ρ	Success rate of incarcerated individuals becoming reformed individuals
10	μ	Natural mortality rate

Furthermore, the diagram in Figure 1 can be expressed as the following mathematical model, which is a nonlinear system

$$S'(t) = \theta\beta - \frac{\alpha S(t)C(t)}{N(t)} - (\mu + v)S(t) + \omega R(t) \tag{1}$$

$$I'(t) = (1 - \theta)\beta + vS(t) - \mu I(t) + \frac{\gamma I(t)R(t)}{N(t)} \tag{2}$$

$$C'(t) = \frac{\alpha S(t)C(t)}{N(t)} - (\mu + \tau + \delta)C(t) \tag{3}$$

$$J'(t) = \delta C(t) - (\mu + \rho)J(t) \tag{4}$$

$$R'(t) = \tau C(t) + \rho J(t) - \frac{\gamma I(t)R(t)}{N(t)} - (\mu + \omega)R(t) \tag{5}$$

The system of fractions from equations (1) – (5) above is expressed in the following system of equations.

$$s'(t) = \theta\beta - \alpha s(t)c(t) - (\mu + v)s(t) + \omega r(t) \tag{6}$$

$$i'(t) = (1 - \theta)\beta + vs(t) - \mu i(t) + \gamma i(t)r(t) \tag{7}$$

$$c'(t) = \alpha s(t)c(t) - (\mu + \tau + \delta)c(t) \tag{8}$$

$$j'(t) = \delta c(t) - (\mu + \rho)j(t) \tag{9}$$

$$r'(t) = \tau c(t) + \rho j(t) - \gamma i(t)r(t) - (\mu + \omega)r(t) \tag{10}$$

In the fraction system (6) – (10) above, the variables in the system of differential equations (1) – (5) are expressed as fractions (percentages) of the total population at a given time. The fractions in equations (6)–(10) above refer to the percentage or proportion of individuals in the population that belong to a particular group. In the context of the fraction system, the fraction variables $[s(t), i(t), c(t), j(t), r(t)]$ describe the percentage of individuals in the population at a given time who belong to the vulnerable, immune, corrupt, incarcerated, or reformist groups. The omission of the parameter N in the fraction system (6)–(10) is intended to simplify analysis and calculations. This is because the model analysis can focus on the fractions of individuals in each group and changes in these fractions over time, without having to consider the total population size separately (Hearne, 2021). By using individual fractions, the proportion of individuals in each group relative to the population can be seen directly. For example, if the fraction of vulnerable individuals, $s(t)$ is 0.6, this means that 60% of the population at that time is vulnerable to corruption.

3.2. Fourth-Order Runge-Kutta Method with Geometric Mean Modification

The fourth-order Runge-Kutta method (RK4) is one of the most widely used numerical methods for solving initial-value problems of ordinary differential equations because it offers a high degree of accuracy and is relatively simple to implement. This method uses four slope values calculated at several points within the integration interval to obtain an approximation of the solution for the next step. In the classical RK4 method, these four slope values are combined using a weighted arithmetic mean, resulting in a fifth-order local error and a fourth-order global error. The classical fourth-order Runge-Kutta method (Classical RK4) is given as follows (Burde, 2011), (Chapra, 2015)

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{11}$$

where

$$k_1 = f(x_n, y_n) \tag{12}$$

$$k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \tag{13}$$

$$k_3 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2) \tag{14}$$

$$k_4 = f(x_n + h, y_n + hk_3) \tag{15}$$

As numerical methods have evolved, various modifications to the Runge-Kutta method have been developed to improve both accuracy and computational efficiency. One such modification is the modified fourth-order Runge-Kutta method based on the geometric mean. This method retains the procedure for calculating the four slope values as in the classical RK4, but replaces the process of combining the slope values with the concept of the geometric mean.

The geometric mean is a measure of central tendency, in addition to the arithmetic mean and the harmonic mean (Bullen, 2003). For two positive real numbers a and b, the geometric mean is defined as

$$GM(a, b) = \sqrt{a \cdot b} \tag{16}$$

In general, for n positive numbers $x_1, x_2, x_3, \dots, x_n$ the geometric mean is given by

$$GM(x_1, x_2, x_3, \dots, x_n) = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \dots x_n} \tag{17}$$

One of the important properties of averages is the inequality relationship among the three types of averages known as the AM-GM-HM (Arithmetic Mean-Geometric Mean-Harmonic Mean) inequality, namely

$$HM \leq GM \leq AM \tag{18}$$

where $M = \frac{\sum_{i=1}^n x_i}{n}$, $GM = \sqrt[n]{\prod_{i=1}^n x_i}$ and $HM = \frac{n \prod_{i=1}^n x_i}{\sum_{i=1}^n x_i}$.

The modified fourth-order Runge–Kutta method based on the geometric mean (RK4GM) was introduced by David J. Evans (Evans, 1991). In his article, Evans replaced the arithmetic mean used in the classical Runge–Kutta formula with the geometric mean, then adjusted the method’s parameters to maintain fourth-order accuracy.

Equation (11) can be written as

$$y_{n+1} = y_n + \frac{1}{3} \left(\frac{k_1 + k_2}{2} + \frac{k_2 + k_3}{2} + \frac{k_3 + k_4}{2} \right)$$

Note that $\frac{k_1+k_2}{2}$ is the arithmetic mean of the numbers k_1 and k_2 . The Runge-Kutta method will be modified according to the geometric mean, so that the arithmetic mean is replaced by the geometric mean, namely $GM(k_1, k_2) = \sqrt{k_1 \cdot k_2}$. If the same is done for k_2, k_3 and k_4 then equation (11) can be written as follows

$$y_{n+1} = y_n + \frac{1}{3} (GM(k_1, k_2) + GM(k_2, k_3) + GM(k_3, k_4)) \tag{19}$$

In the geometric mean-modified fourth-order Runge–Kutta method, the values of k_1, k_2, k_3 and k_4 are calculated using the same procedure as in the classical RK4 method. According to (Wazwaz, 1994), the values of k_1, k_2, k_3 and k_4 are

$$k_1 = f(x_n, y_n) \tag{20}$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) \tag{21}$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{16}(-k_1 + 9k_2)\right) \tag{22}$$

$$k_4 = f\left(x_n + h, y_n + \frac{h}{24}(-3k_1 + 5k_2 + 22k_3)\right) \tag{23}$$

The advantage of this method lies in its ability to provide an alternative approach to the classical RK4 method by utilizing the multiplicative property of the geometric mean. In some cases of differential equations, particularly those with significant changes in the gradient, this method can produce smaller errors compared to methods that use the arithmetic mean. Furthermore, the number of function evaluations required remains the same as in the classical RK4 method, namely four function evaluations at each integration step.

However, the application of the geometric mean-modified fourth-order Runge–Kutta method requires special attention because the geometric mean is only defined for values that do not result in a negative product. Therefore, the characteristics of the differential equation to be solved must be considered so that the method can be applied correctly and produce a stable numerical solution.

The 4th-order Runge-Kutta algorithm with harmonic averaging modification (Lambert, 1973), (Buthcer, 1987)

1. Given the initial values x_0, y_0 , the step size h , and the number of iterations
2. Find

$$\begin{aligned} k_1 &= f(x_n, y_n) \\ k_2 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) \\ k_3 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{16}(-k_1 + 9k_2)\right) \\ k_4 &= f\left(x_n + h, y_n + \frac{h}{24}(-3k_1 + 5k_2 + 22k_3)\right) \end{aligned}$$

3. Calculate the geometric mean

$$\begin{aligned} GM(k_1, k_2) &= \sqrt{k_1 \cdot k_2} \\ GM(k_2, k_3) &= \sqrt{k_2 \cdot k_3} \end{aligned}$$

$$GM(k_3, k_4) = \sqrt{k_3 \cdot k_4}$$

4. Find

$$y_{n+1} = y_n + \frac{1}{3}(GM(k_1, k_2) + GM(k_2, k_3) + GM(k_3, k_4))$$

5. Repeat the process until you reach the end of the interval.

3.3. Application of the Geometric Mean-Modified Fourth-Order Runge-Kutta Method (RK4GM) to a Mathematical Model of the Dynamics of Corruption in Indonesia

In this study, the numerical solution to the system of differential equations (6–10) will be determined using the fourth-order Runge-Kutta method with geometric mean modification (RK4GM). According to Equation (19), the numerical solution to the system (6–10) is

$$s_{n+1} = s_n + \frac{1}{3}(\sqrt{k_1 \cdot k_2} + \sqrt{k_2 \cdot k_3} + \sqrt{k_3 \cdot k_4})$$

$$i_{n+1} = i_n + \frac{1}{3}(\sqrt{l_1 \cdot l_2} + \sqrt{l_2 \cdot l_3} + \sqrt{l_3 \cdot l_4})$$

$$c_{n+1} = c_n + \frac{1}{3}(\sqrt{m_1 \cdot m_2} + \sqrt{m_2 \cdot m_3} + \sqrt{m_3 \cdot m_4})$$

$$j_{n+1} = j_n + \frac{1}{3}(\sqrt{n_1 \cdot n_2} + \sqrt{n_2 \cdot n_3} + \sqrt{n_3 \cdot n_4})$$

$$r_{n+1} = r_n + \frac{1}{3}(\sqrt{p_1 \cdot p_2} + \sqrt{p_2 \cdot p_3} + \sqrt{p_3 \cdot p_4})$$

where $k_i, l_i, m_i, n_i,$ and p_i follow Equations (20–23) and correspond, respectively, to systems (6–10).

Applying the RK4GM method to a system of differential equations (6–10) requires fairly complex calculations because each variable in the system must be calculated simultaneously at every iteration step. The complexity of these calculations makes manual solution impractical, especially when the observation time interval is quite long or the step size used is relatively small. A large number of iterations not only requires a long computation time but also increases the likelihood of computational errors. Therefore, numerical simulation using computational software is necessary for effective implementation (Matthes, 2023), (Downey, 2024).

In this study, numerical simulations were performed using the Python programming language. Python was chosen for its ability to perform numerical computations efficiently and for its ease in processing and visualizing simulation data. Through the implementation of the RK4GM algorithm, numerical solutions were obtained for each variable in the corruption behavior dynamics model, allowing the population dynamics in each compartment to be observed over time (Langtagen, 2016) (Kong, 2021).

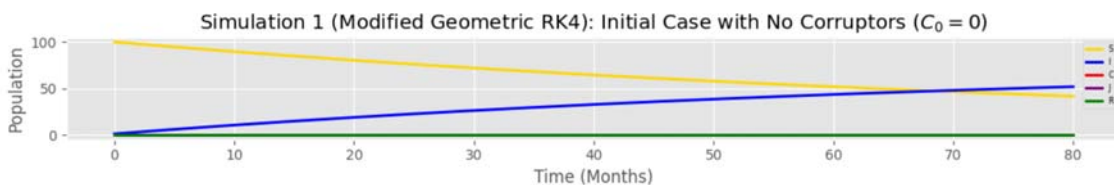
The following is a numerical simulation of the RK4GM method on System (6-10):

The parameter values in Table 1 are taken as follows:

$$\theta = 1, \beta = 0,001, \alpha = 0,003, \gamma = 0,0001, v = 0,01, \omega = 0,0001, \delta = 0,05, \tau = 0,05, \rho = 0,05, \mu = 0,001$$

Simulation 1:

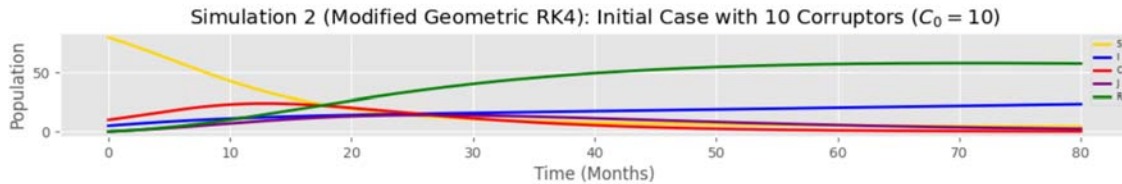
Given the initial conditions $s_o = 100, i_o = 1, c_o = 0, j_o = 0,$ and $r_o = 0$



These results show that when there are no corrupt individuals at the outset, corrupt behavior does not develop within the system. The population tends to move toward the group with integrity (I) without the emergence of new cases of corruption. Thus, this scenario illustrates an ideal situation in which the spread of corrupt behavior can be prevented from the outset.

Simulation 2:

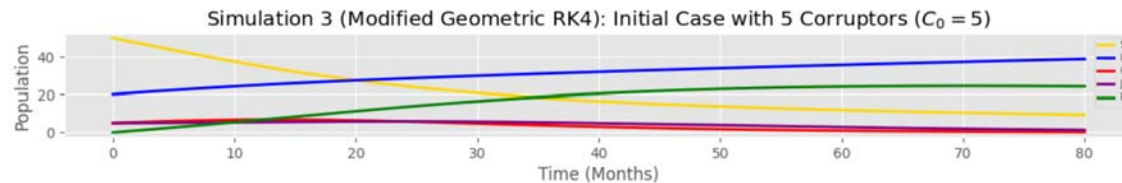
Given the initial conditions $s_o = 80, i_o = 5, c_o = 10, j_o = 0$, dan $r_o = 0$



This phenomenon shows that although corruption initially increased, the model's oversight, enforcement, and remediation mechanisms were able to reduce the number of corrupt individuals, eventually bringing it down to zero. Most individuals then moved to Group R, which can be interpreted as individuals who have been deterred, have become law-abiding, or have successfully broken free from corrupt behavior.

Simulation 3:

Given the initial conditions $s_o = 50, i_o = 20, c_o = 5, j_o = 5$, dan $r_o = 0$



These results show that a smaller initial number of corrupt officials leads to a more controlled spread of corruption compared to the second simulation. The system reaches a stable state more quickly with a very low number of corrupt officials.

4. CONCLUSION

This study successfully developed a mathematical model of the dynamics of corrupt behavior in Indonesia in the form of a system of differential equations capable of representing the interactions among population groups involved in the emergence, development, and decline of corrupt behavior. The model provides a mathematical description of changes in the number of individuals in each compartment over time, thereby enabling a more systematic understanding of the dynamics of corrupt behavior.

Since the resulting system of differential equations is difficult to solve analytically, this study applies a fourth-order Runge–Kutta method with a geometric mean modification to obtain its numerical solution. The results of the implementation show that this method can be used effectively to solve the developed system of differential equations. The use of the geometric mean in the slope approximation process yields a numerical procedure capable of accurately tracking the system's dynamics at each calculation step.

The numerical simulations conducted using Python successfully depicted the behavior of the solutions from the corruption dynamics model for various observation periods. Through

these simulation results, the evolution of each variable in the model can be observed quantitatively, thereby providing information on trends in changes in corruption behavior within the studied population. In addition to simplifying complex computational processes, numerical simulations also allow for the visualization of system dynamics, making it easier to interpret the model's behavior.

Thus, it can be concluded that the development of a mathematical model of corruption dynamics in Indonesia, along with the application of the fourth-order Runge–Kutta method modified by the geometric mean, has been successfully carried out. This combination of mathematical modeling and numerical simulation provides an effective approach for quantitatively analyzing the phenomenon of corruption and can serve as a foundation for further research into the development of more accurate models and numerical methods for studying the dynamics of corruption in Indonesia.

One of the open questions arising from this study is the further development of mathematical models that can account for additional factors—such as political, economic, and cultural dynamics—that also influence the spread of corruption in Indonesia. Although the current model is quite comprehensive in dividing the population into various compartments, it still needs to be refined to accommodate the complexity of human and institutional behavior, which cannot be fully explained by a simple deterministic model. Furthermore, these models have not yet fully captured the impact of long-term legal reforms and anti-corruption policies, nor the more complex interactions among individuals within social networks. Developing models capable of predicting the long-term effects of various intervention strategies, while accounting for uncertainty and variability in social parameters, remains an intriguing challenge for future research.

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