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APPLICATION OF INTEGRALS IN DETERMINING THE MOMENT OF INERTIA OF A TRIANGLE PLANE WITH RESPECT TO A LINE THROUGH THE CENTROID OF THE TRIANGLE

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Abstract

Moment of Inertia is a measure of the inertia of an object to rotate about its axis. In this research, we will discuss determining the moment of inertia of a triangular plane whose axis is at the centroid point of the triangle. The plane can be a right triangle, isosceles triangle or non-isosceles triangle.

Determining the moment of inertia of a plane can theoretically be done by applying the concept of a certain integral. The work process begins by first determining the coordinates of the centroid point of the triangle and then determining the boundaries of a certain integral which will then produce the magnitude of the moment of inertia of the plane. The results of this research are the magnitude of the moment of inertia of the triangular plane regarding the x-axis and the moment of inertia of the triangular plane regarding the y-axis which has its axis at the centroid point of the triangle.

Keywords: Triangular Centroid Point, Triangular Moment of Inertia, Triangular Centroid Point Coordinates.

1. INTRODUCTION

One of the interesting discussions about triangles is about the center of the triangle. The center point of this triangle has many applications, one of which is the centroid which can be applied as the axis of rotation of an object. Objects that are in rotational motion have a tendency to maintain their rotational motion. The tendency of an object to maintain its rotational motion is called the moment of inertia. The moment of inertia can be defined as the product of mass by the squared distance from the axis. The moment of inertia can also be said to be a measure of the inertia of an object to rotate about its axis. The rotation axis of the plane can be located at the origin or at the center of mass. If the piece is in the shape of a triangular plane then the center of mass is the centroid point so that the moment of inertia pivoting at the triangular centroid point can also be determined both about the x axis and the y-axis.

2. TRIANGLE

A triangle is a flat shape that has three sides. The type of triangle can be seen from the length of its sides or the size of the angles inside. If we look at the length of the sides, triangles can be classified as equilateral triangles, isosceles triangles and arbitrary triangles. If we look at the size of the angles in a triangle, they can be classified as acute triangles, right triangles and obtuse triangles.

Definition 1. Triangle Weight Line (Balmer, 2010)
Given a triangle ABC as in Figure 1. The bisector AD is the line connecting point A and point D where point D is the midpoint BC.

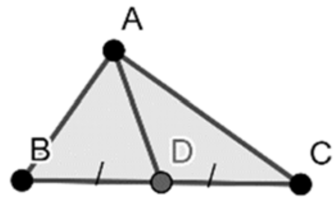


Figure 1. Triangle Weight Line

Definition 2. Centroid Point (Chernega et al., 2017a)
The centroid point is the intersection point of the three bisectors of triangle ABC, as in Figure 2.

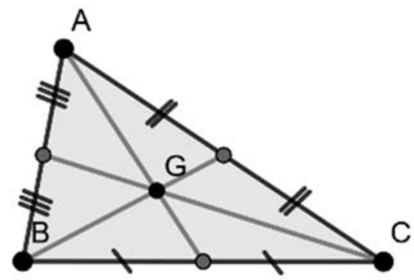
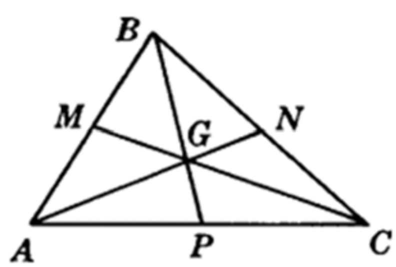


Figure 2. Centroid in Triangle ABC

Principle 3. Triangular Centroid (Chernega, et al., 2017b)
If a triangle ABC is given and point G is the centroid of triangle ABC, then $AG: GN = 2:1, BG : GP = 2:1$ and $CG: GM = 2:1$.



Gambar 3. Titik *Centroid* dalam Segitiga ABC

Theorem 4. Triangular Centroid Coordinates (Hatzipolakis & Yiu, 2009)
Given a triangle with vertices $A(x_a, y_a)$, $B(x_b, y_b)$ and $C(x_c, y_c)$. If three bisectors are drawn in triangle ABC, Figure 4, which intersect at one centroid point, namely point G, then the coordinates of the centroid point of triangle ABC are

$$\begin{aligned} G(x_g, y_g) = \big(&[(x_a x_b - {x_c}^2)(y_a - y_b) + (x_a x_c - {x_b}^2)(y_c - y_a) \\ &+ (x_b x_c - {x_a}^2)(y_b - y_c)] \\ &/\big[3\big(x_a(y_c - y_b) + x_b(y_a - y_c) + x_c(y_b - y_a)\big)\big]\big), \big([(y_a y_b - {y_c}^2)(x_a \\ &- x_b) + (y_a y_c - {y_b}^2)(x_c - x_a) + (y_b y_c - {y_a}^2)(x_b - x_c)] \\ &/\big[3\big(y_a(x_c - x_b) + y_b(x_a - x_c) + y_c(x_b - x_a)\big)\big]\big) \end{aligned}$$

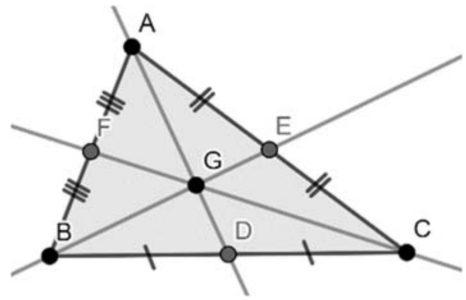


Figure 4. Centroid Point in Triangle ABC

Example 5. Centroid point coordinates (Nagle et al., 2014)
Look at Triangle ABC Figure 5.

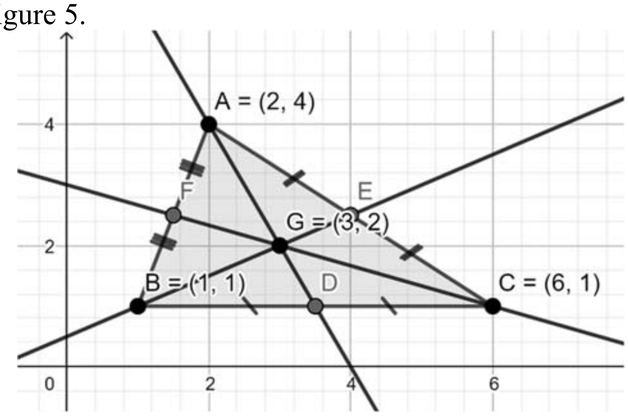


Figure 5. Centroid Point in Triangle ABC

The coordinates of the centroid point G are

$$x_g = \frac{(2.1 - 6^2)(4 - 1)(2.6 - 1^2)(1 - 4) + (1.6 - 2^2)(1 - 1)}{3\big(2(1 - 1) + 1(4 - 1) + 6(1 - 4)\big)}$$
$$= \frac{(-34).3 + 11(-3) + 2.0}{3\big(2.0 + 3 + 6(-3)\big)}$$
$$= \frac{(-135)}{(-45)} = 3$$
$$y_g = \frac{(4.1 - 1^2)(2 - 1) + (4.1 - 1^2)(6 - 2) + (1.1 - 4^2)(1 - 6)}{3\big(4(6 - 1) + 1(2 - 6) + 1(1 - 2)\big)}$$
$$= \frac{3.1 + 3.4 + (-15)(-5)}{3\big(4.5 + 1.(-4) + 1(-1)\big)}$$
$$= \frac{90}{45} = 2,$$

so that the centroid coordinate point is $G(3,2)$.

3. MOMENT OF INERTIA

The moment of inertia or what is usually called the moment of inertia is a measurement that shows how much an object tends to maintain its condition. In mechanics, mass is seen as an object that is concentrated at a point called the center of mass (center of gravity). For a homogeneous object, this point coincides with its geometric center or center of gravity, so that in a triangular plane, the center of mass is the centroid of the triangle.

The moment of inertia of a plane about line L can be determined by following the steps below.

(1) Sketch the area and a rectangular band parallel to the L line.
(2) Determine the product of the area of the tape and the square of the distance from the centroid of the tape to line L.
(3) Calculating the moment of inertia of an area using an integral (Wildberger, 2011).

Definition 6. Moment of Inertia (Ayres, 1985)
(a) The moment of inertia of a plane about the x-axis is defined in the following integral form.

$$I_x = \int_{y_1}^{y_2} y^2 dA$$

(b) The moment of inertia of a plane about the y-axis is defined in the following integral form.

$$I_y = \int_{x_1}^{x_2} x^2 dA$$

where dA is the area of the rectangular band parallel to the rotation axis.

4. MOMENT OF INERTIA OF A PLANE REGARDING A LINE THROUGH THE CENTROID OF A TRIANGLE

The moment of inertia of a triangular plane about the line L passing through the centroid can be determined by following the steps below.
(1) Make a sketch of the area that describes the triangular area.
(2) Determine the centroid point of the triangle and draw a line L (*x* or *y* axis) through the centroid point.
(3) Make a rectangular ribbon parallel to the L line (*x* or *y* axis).
(4) Determine the product of the area of the tape and the square of the distance from the centroid of the tape to line L.
(5) Calculating the moment of inertia of the plane using integration (Yiu, 2004).

4.1 Right triangle

(a) Moment of inertia about the *x* axis (I_x)

The moment of inertia of a right triangle about the *x* axis passing through the centroid of the triangle is illustrated in the following figure.

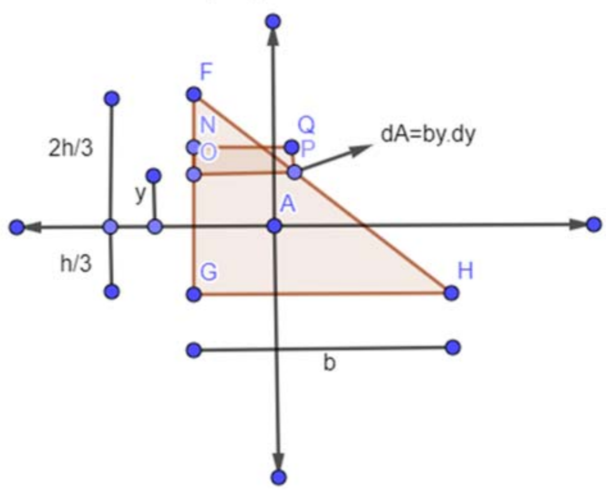


Figure 6. Illustration of the moment of inertia of a right-angled triangle about the *x*-axis

Information:
(1) Point A is the centroid point of the right triangle FGH

(2) The NOPQ rectangle represents the band parallel to the *x*-axis
(3) y is the distance from the center of gravity of the tape to the *x*-axis

The steps to determine the moment of inertia of a triangular plane are as follows.
(1) Draw a plane in the form of a right triangle with base length b and height h.
(2) Determine the centroid point of the triangle using Principle 3.
(3) Make the *x*-axis and *y*-axis intersect at the centroid point of the triangle.
(4) Make a rectangular ribbon parallel to the *x*-axis.
(5) Determine the area of the dA band.
(6) Determine the boundaries of the triangular plane perpendicular to the *x*-axis.
(7) Calculate the moment of inertia using a certain integral of the area of the tape multiplied by the square of the distance from the center of gravity of the tape to the *x*-axis.

The moment of inertia of a right triangle with base length *b* and height *h* is as follows.

$$I_x = \int_{y_1}^{y_2} y^2 dA = \int_{-h/3}^{2h/3} y^2 b_y dy.$$

Because the value of b_y is different for each y, the value of b_y can be determined by an equation in y which satisfies the condition of length $b_y = 0$ if $y = {}^{2h}/_{3}$ and $b_y = b$ if $y = -{}^{h}/_{3}$. So the value of b_y is

$$\frac{b_y - 0}{b - 0} = \frac{y - {}^{2h}/_{3}}{-{}^{h}/_{3} - {}^{2h}/_{3}}$$

or

$$b_y = b\left(\frac{2}{3} - \frac{y}{h}\right),$$

so the moment of inertia of the area about the *x*-axis is

$$\begin{aligned} I_x &= \int_{-h/3}^{2h/3} y^2 b\left(\frac{2}{3} - \frac{y}{h}\right) dy \\ &= \int_{-h/3}^{2h/3} \left(\frac{2b}{3} y^2 - \frac{by^3}{h}\right) dy \\ &= \left(\frac{2b}{9} y^3 - \frac{b}{4h} y^4\right)_{-h/3}^{2h/3} = \frac{bh^3}{36}. \end{aligned}$$

(b) Moment of inertia about the *y*-axis (I_y)

The moment of inertia of a right triangle about the y axis passing through the centroid of the triangle is illustrated in the following figure.

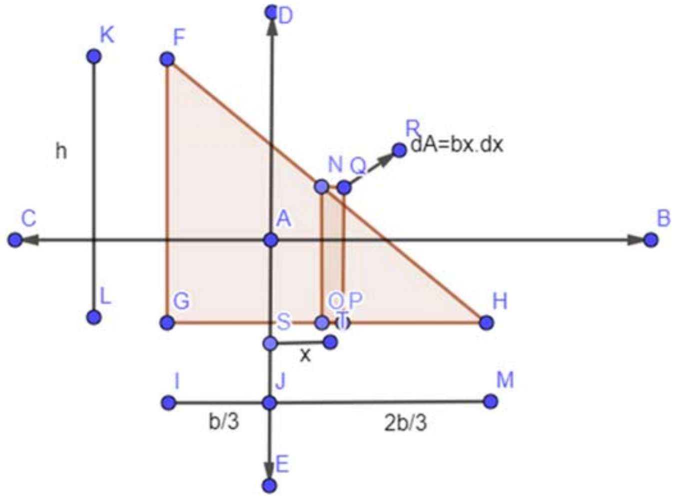


Figure 7. Illustration of the moment of inertia of a right-angled triangle about the *y*-axis

Information:
(1) The *x*-axis is depicted as CB and the *y*-axis is depicted as DE.
(2) Point A is the centroid point of the right triangle FGH.
(3) The NOPQ rectangle represents the band parallel to the y-axis.
(4) *x* is the distance from the center of gravity of the tape to the *y*-axis.

The steps to determine the moment of inertia of a triangular plane are as follows.
(1) Draw a plane in the form of a right triangle with base length b and height h.
(2) Determine the centroid point of the triangle using Principle 3.
(3) Make the x-axis and y-axis intersect at the centroid point of the triangle.
(4) Make a rectangular ribbon parallel to the *y*-axis.
(5) Determine the area of the dA band.
(6) Determine the boundaries of the triangular plane perpendicular to the y-axis.
(7) Calculate the moment of inertia using a certain integral of the area of the tape multiplied by the square of the distance from the center of gravity of the tape to the y-axis.

The moment of inertia of a right triangle with base length *b* and height *h* is as follows.

$$I_y = \int_{x_1}^{x_2} x^2 dA = \int_{-b/3}^{2b/3} x^2 b_x dx$$

Because the value of b_x is different for each *x*, the value of b_x can be determined by an equation in *x* which satisfies the length condition panjang $b_x = 0$ if $x = {}^{2b}/_{3}$ and $b_x = h$ if $= -{}^{b}/_{3}$. So the value of b_x is

$$\frac{b_x - 0}{h - 0} = \frac{x - {}^{2b}/_{3}}{-{}^{b}/_{3} - {}^{2b}/_{3}}$$

or

$$b_x = h\left(\frac{2}{3} - \frac{x}{b}\right),$$

so the moment of inertia of the area about the *y*-axis is

$$I_y = \int_{-b/3}^{2b/3} x^2 h\left(\frac{2}{3} - \frac{x}{b}\right) dx$$

$$= \int_{-b/3}^{2b/3} \left(\frac{2h}{3} x^2 - \frac{hx^3}{b} \right) dx$$

$$= \left(\frac{2h}{9} x^3 - \frac{h}{4b} x^4 \right)_{-b/3}^{2b/3} = \frac{hb^3}{36}.$$

4.2. Isosceles Triangle

(a) Moment of inertia about the x axis (I_x)

The moment of inertia of an isosceles triangle about the x-axis passing through the centroid of the triangle is illustrated in the following figure.

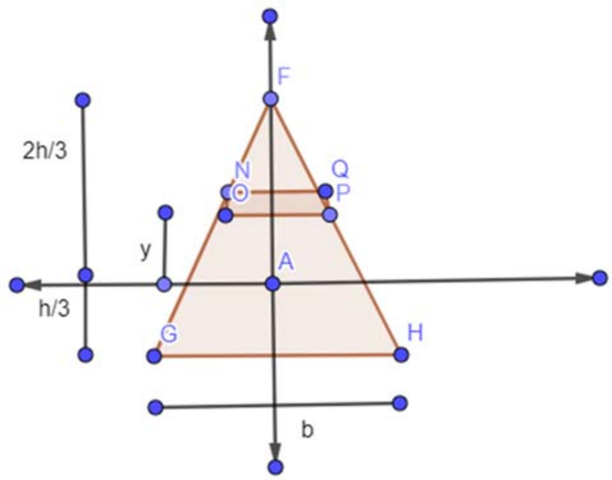


Figure 8. Illustration of the moment of inertia of an isosceles triangle about the x-axis

Information:
(1) Point A is the centroid point of the isosceles triangle FGH.
(2) The NOPQ rectangle represents the band parallel to the x-axis.
(3) y is the distance from the center of gravity of the tape to the x-axis.

The steps to determine the moment of inertia of a triangular plane are as follows.
(1) Draw a plane in the form of an isosceles triangle with base length b and height h.
(2) Determine the centroid point of the triangle using Principle 3.
(3) Make the x-axis and y-axis intersect at the centroid point of the triangle.
(4) Make a rectangular ribbon parallel to the x-axis.
(5) Determine the area of the dA band.
(6) Determine the boundaries of the plane perpendicular to the x-axis.
(7) Calculate the moment of inertia using a certain integral of the area of the tape multiplied by the square of the distance from the center of gravity of the tape to the x-axis.

The moment of inertia of an isosceles triangle with base length b and height h is as follows.

$$I_x = \int_{y_1}^{y_2} y^2 dA = \int_{-h/3}^{2h/3} y^2 b_y dy.$$

Because the value of b_y is different for each y, the value of b_y can be determined by an equation in y which satisfies the condition of length $b_y = 0$ if $y = {}^{2h}/_{3}$ and $b_y = b$ if $= -{}^{h}/_{3}$. So the value of b_y is

$$\frac{b_y - 0}{b - 0} = \frac{y - {}^{2h}/_{3}}{-{}^{h}/_{3} - {}^{2h}/_{3}}$$

or

$$b_y = b\left(\frac{2}{3} - \frac{y}{h}\right),$$

so the moment of inertia of the area about the *x*-axis is

$$\begin{aligned} I_x &= \int_{-h/3}^{2h/3} y^2 b\left(\frac{2}{3} - \frac{y}{h}\right) dy \\ &= \int_{-h/3}^{2h/3} \left(\frac{2b}{3}y^2 - \frac{by^3}{h}\right) dy \\ &= \left(\frac{2b}{9}y^3 - \frac{b}{4h}y^4\right)_{-h/3}^{2h/3} = \frac{bh^3}{36}. \end{aligned}$$

(b) Moment of inertia about the *y*-axis (I_y)

The moment of inertia of an isosceles triangle about the *y*-axis passing through the centroid of the triangle is illustrated in the following figure.

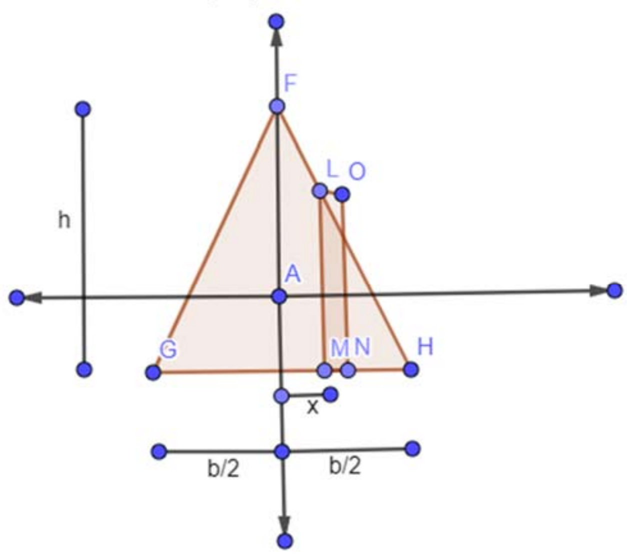


Figure 9. Illustration of the moment of inertia of an isosceles triangle about the *y*-axis

Information:
(1) Point A is the centroid point of the isosceles triangle FGH.
(2) The LMNO rectangle represents the band parallel to the *y*-axis.
(3) *x* is the distance from the center of gravity of the tape to the *y*-axis.

The steps to determine the moment of inertia of a triangular plane are as follows.
(1) Draw a plane in the form of an isosceles triangle with base length *b* and height *h*.
(2) Determine the centroid point of the triangle using Principle 3.
(3) Make the *x*-axis and *y*-axis intersect at the centroid point of the triangle.
(4) Make a rectangular ribbon parallel to the *y*-axis.
(5) Determine the area of the dA band.
(6) Determine the boundaries of the plane perpendicular to the *y*-axis.

(7) Calculate the moment of inertia using a certain integral of the area of the tape multiplied by the square of the distance from the center of gravity of the tape to the y-axis.

The moment of inertia of an isosceles triangle with base length b and height h is as follows.

$$I_y = \int_{x_1}^{x_2} x^2 dA = \int_{-b/2}^{b/2} x^2 b_x dx$$

Because the value of b_x is different for each x, the value of b_x can be determined by an equation in x that satisfies the length conditions $b_x = 0$ if $x = b/2$ and $b_x = h$ if $x = 0$. So the value of b_x is

$$\frac{b_x - 0}{h - 0} = \frac{x - b/2}{0 - b/2}$$

or

$$b_x = h - \frac{2h}{b}x,$$

so the moment of inertia of the area about the y-axis is

$$\begin{aligned} I_y &= \int_{x_1}^{x_2} x^2 dA = \int_{-b/2}^{b/2} x^2 b_x dx \\ &= 2\int_0^{b/2} x^2\left(h - \frac{2h}{b}x\right)dx \\ &= 2\int_0^{b/2}\left(hx^2 - \frac{2h}{b}x^3\right)dx \\ &= 2.\left(\frac{h}{3}x^3 - \frac{h}{2b}x^4\right)_0^{b/2} = \frac{hb^3}{48}. \end{aligned}$$

4.3. The triangle is not isosceles

(a) Moment of inertia about the x-axis (I_x)

The moment of inertia of an unequal triangle about the x-axis passing through the centroid of the triangle is illustrated in the following figure.

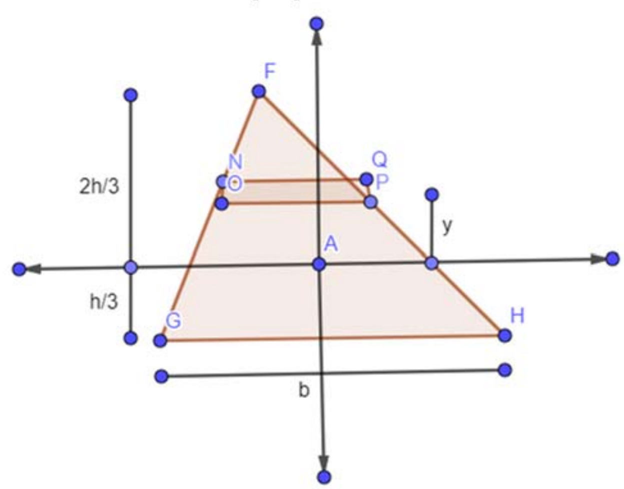


Figure 10. Illustration of the moment of inertia of an unequal triangle with respect to the x-axis

Information:
(1) Point A is the centroid point of triangle FGH.
(2) The NOPQ rectangle represents the band parallel to the *x*-axis.
(3) y is the distance from the center of gravity of the tape to the *x*-axis.

The steps to determine the moment of inertia of a triangular plane are as follows:
(1) Draw a plane in the shape of an isosceles triangle with base length b and height h.
(2) Determine the centroid point of the triangle using Principle 3.
(3) Make the *x*-axis and y-axis intersect at the centroid point of the triangle.
(4) Make a rectangular ribbon parallel to the *x*-axis.
(5) Determine the area of the dA band.
(6) Determine the boundaries of the plane perpendicular to the *x* axis.
(7) Calculate the moment of inertia using a certain integral of the area of the tape multiplied by the square of the distance from the center of gravity of the tape to the *x*-axis.

The moment of inertia of a non-isosceles triangle with base length b and height h is as follows.

$$I_x = \int_{y_1}^{y_2} y^2 dA = \int_{-h/3}^{2h/3} y^2 b_y dy.$$

Because the value of b_y is different for each y, the value of b_y can be determined by an equation in y which satisfies the condition of length $b_y = 0$ if $y = {}^{2h}/_{3}$ and $b_y = b$ if $y = -{}^{h}/_{3}$. So the value of b_y is

$$\frac{b_y - 0}{b - 0} = \frac{y - {}^{2h}/_{3}}{-{}^{h}/_{3} - {}^{2h}/_{3}}$$

or

$$b_y = b\left(\frac{2}{3} - \frac{y}{h}\right),$$

so the moment of inertia of the area about the *x*-axis is

$$\begin{aligned} I_x &= \int_{-h/3}^{2h/3} y^2 b\left(\frac{2}{3} - \frac{y}{h}\right) dy \\ &= \int_{-h/3}^{2h/3} \left(\frac{2b}{3} y^2 - \frac{by^3}{h}\right) dy \\ &= \left(\frac{2b}{9} y^3 - \frac{b}{4h} y^4\right)_{-h/3}^{2h/3} = \frac{bh^3}{36}. \end{aligned}$$

(b) Moment of inertia about the *y*-axis (I_y)

The moment of inertia of an unequal triangle about the y-axis passing through the centroid of the triangle is illustrated in the following figure.

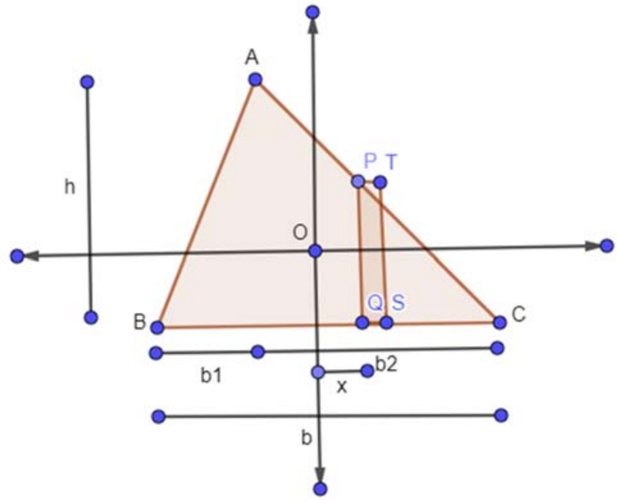


Figure 11. Illustration of the moment of inertia of an unequal triangle with respect to the y-axis

Information:

(1) Point O is the centroid point of triangle ABC with coordinates $A(x_a, y_a)$, $B(x_b, y_b)$ and $C(x_c, y_c)$.

(2) The PQST rectangle represents the band parallel to the y-axis.

(3) x is the distance from the center of gravity of the tape to the y-axis.

The steps to determine the moment of inertia of a triangular plane are as follows.

(1) Draw a plane in the shape of an isosceles triangle with base length b and height h.

(2) Determine the centroid point of the triangle using Principle 3.

(3) Make the x-axis and y-axis intersect at the centroid point of the triangle.

(4) Make a rectangular ribbon parallel to the y-axis.

(5) Determine the area of the band dA.

(6) Determine the boundaries of the plane perpendicular to the y-axis

(7) Calculate the moment of inertia using a certain integral of the area of the tape multiplied by the square of the distance from the center of gravity of the tape to the y-axis

The moment of inertia of a non-isosceles triangle with base length b and height h is as follows.

$$I_y = \int_{x_1}^{x_2} x^2 dA = \int_{-b_1}^{b_2} x^2 b_x dx = \int_{-b_1}^{x_a} x^2 b_x dx + \int_{x_a}^{b_2} x^2 b_x dx$$

Because the value of b_x is different for each x, the value of b_x can be determined in the following way.

(1) If you pay attention to the area to the left of the line $x = x_a$ then the length $b_x =$ if $x = -b_1$ and $b_x = h$ if $x = x_a$. So the value of b_x is

$$\frac{b_x - 0}{h - 0} = \frac{x - (-b_1)}{x_a - (-b_1)}$$

or

$$b_x = \frac{h(x + b_1)}{x_a + b_1},$$

so that the moment of inertia of the area to the left of the line $x = x_a$ about the y-axis is

$$\int_{-b_1}^{x_a} x^2 b_x dx = \int_{-b_1}^{x_a} x^2 \frac{h(x+b_1)}{x_a+b_1} dx$$
$$= \frac{h}{x_a+b_1}\int_{-b_1}^{x_a}(x^3+b_1x^2)dx$$
$$= \frac{h}{x_a+b_1}\left(\frac{1}{4}x^4+\frac{b_1}{3}x^3\right)_{-b_1}^{x_a}$$
$$= \frac{h}{x_a+b_1}\left(\frac{1}{4}{x_a}^4+\frac{b_1}{3}{x_a}^3+\frac{1}{12}{b_1}^4\right).$$

(2) If you pay attention to the area to the right of the line $x = x_a$ then the length $b_x = h$ if $x = x_a$ and $b_x = 0$ if $x = b_2$. So the value of b_x is

$$\frac{b_x - h}{0-h} = \frac{x - x_a}{b_2 - x_a}$$

or

$$b_x = \frac{h(b_2 - x)}{b_2 - x_a},$$

so that the moment of inertia of the area to the right of the line $x = x_a$ about the *y*-axis is

$$\int_{x_a}^{b_2} x^2 b_x dx = \int_{x_a}^{b_2} x^2 \frac{h(b_2-x)}{b_2-x_a} dx$$
$$= \frac{h}{b_2-x_a}\int_{x_a}^{b_2}(b_2x^2-x^3)dx$$
$$= \frac{h}{b_2-x_a}\left(\frac{b_2}{3}x^3-\frac{1}{4}x^4\right)_{x_a}^{b_2}$$
$$= \frac{h}{b_{2-}x_a}\left(\frac{1}{4}{x_a}^4-\frac{b_2}{3}{x_a}^3+\frac{1}{12}{b_2}^4\right).$$

The moment of inertia of this area about the *y*-axis is

$$I_y = \int_{-b_1}^{x_a} x^2 b_x dx + \int_{x_a}^{b_2} x^2 b_x dx$$
$$= \frac{h}{x_a+b_1}\left(\frac{1}{4}{x_a}^4+\frac{b_1}{3}{x_a}^3+\frac{1}{12}{b_1}^4\right) + \frac{h}{b_{2-}x_a}\left(\frac{1}{4}{x_a}^4-\frac{b_2}{3}{x_a}^3+\frac{1}{12}{b_2}^4\right).$$

3. CONCLUSION

From the description above, it can be concluded:

(1) The coordinates of the centroid point G in triangle ABC with $A(x_{a,}y_a), B(x_{b,}y_b)$, and $C(x_{c,}y_c)$ are

$$G(x_g, y_g) = \Big([(x_ax_b - {x_c}^2)(y_a - y_b) + (x_ax_c - {x_b}^2)(y_c - y_a) + (x_bx_c - {x_a}^2)(y_b - y_c)] / \big[3\big(x_a(y_c - y_b) + x_b(y_a - y_c) + x_c(y_b - y_a)\big)\big]\Big), \Big([(y_ay_b - {y_c}^2)(x_a - x_b) + (y_ay_c - {y_b}^2)(x_c - x_a) + (y_by_c - {y_a}^2)(x_b - x_c)] / \big[3\big(y_a(x_c - x_b) + y_b(x_a - x_c) + y_c(x_b - x_a)\big)\big]\Big)$$

(2) (a) The Moment of Inertia of a plane in the shape of a right triangle with base *b* and height *h* with respect to the *x*-axis passing through the centroid point is $I_x = \frac{bh^3}{36}$.

(b) The Moment of Inertia of a plane in the shape of a right triangle with base *b* and height *h* with respect to the *y*-axis passing through the centroid point is $I_y = \frac{hb^3}{36}$.

(3) (a) The Moment of Inertia of an isosceles triangular plane with base b and height h about the x-axis passing through the centroid point is $I_x = \frac{bh^3}{36}$.

(b) The Moment of Inertia of an isosceles triangular plane with base b and height h with respect to the y-axis passing through the centroid point is $I_y = \frac{hb^3}{48}$.

(4) (a) The Moment of Inertia of a non-isosceles triangular plane with base b and height h with respect to the x-axis passing through the centroid point is $I_x = \frac{bh^3}{36}$.

(b) The moment of inertia of a plane in the shape of an unequal triangle t with base b and height h with respect to the y-axis passing through the centroid point is

$$I_y = \frac{h}{x_a + b_1}\left(\frac{1}{4}{x_a}^4 + \frac{b_1}{3}{x_a}^3 + \frac{1}{12}{b_1}^4\right) + \frac{h}{b_{2-}x_a}\left(\frac{1}{4}{x_a}^4 - \frac{b_2}{3}{x_a}^3 + \frac{1}{12}{b_2}^4\right)$$

where

x_a is the abscissa of vertex A

b_1 is the distance from the abscissa of point B to the abscissa of vertex A

b_2 is the distance from the abscissa of point C to the abscissa of vertex A.

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