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Subdivision of a Regular Polygon

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Abstract: The basic principle and techniques discussed in this research belongs to combinatorial analysis. Motivated by the problem in the international mathematics Olympiad of which, as written in, Lin Ziwei, a participant in the international mathematics Olympiad, age 14 then, was able to solve the problem of subdivision of a triangle into parallelogram with an hour. The researcher feels it is challenging to extend a concept of nth subdivision of triangle into subdivision of a regular polygon. In this research, results obtained include the explicit formula for giving the number of triangles in nth subdivision of any regular polygon and several illustrations leading into a conjecture on the non-existence of restricted polygons in the nth subdivision.

Keywords: regular polygon, subdivision.

Introduction

A Regular Polygon can be subdivided into parts in so many ways. The problem, which for instance in [1] about subdividing an equilateral triangle into n+1 equal parts was given to the trainees in the 1990 Singapore Mathematical Olympiad Team as a test problem on June 15, 1989. The main question then was to derive a formula on the number of parallelogram of the nth subdivision of an equilateral triangle. Lin Ziwei, a member in the team, of age 14 then, was able to solve the problem within an hour.

Subdivision has become a staple of the geometric modeling community allowing coarse, polygon shapes to represented highly refined, smooth shape with guaranteed continuity properties. It is a powerful and easily implemented algorithm used, in its simplest application to smooth meshes. The underlying concepts are derived from spline refinement algorithms, but the idea is that there exists a well-defined smooth surface associated with any given input mesh (the exact surface depends on the division algorithm used.) Depending on the type of input mesh (triangular, quadrilateral, etc.) We must used a different subdivision algorithms exist, and within triangular meshes, there is still a wide array, although some are visually superior to others. Quadrilateral-based meshes generally use Catmull-Clark, while triangular-based meshes generally use Loop subdivision.

Subdivision surfaces are currently one of themost powerful surface representations used to model smooth shapes. Subdivision surface is a very popular geometric modeling method, which can be used to create smooth surfaces with arbitrary topological structures. It has the advantages of strong numerical stability and high execution efficiency. It has been successfully applied to computer graphics, films and television, animation, and other fields.

The visual quality of a subdivision surface depends in a crucial way on the initial, or base, mesh of control vertices. Subdivision surfaces were introduced in 1978 by both Catmull and Clark and Doo and Sabin.

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Results and Discussion

THE n^{th} SUBDIVISION OF REGULAR POLYGON

We first make a generalized version of Definition 1.6.1. For $n \in \mathbb{N}$, the n^{th} subdivision of a polygon with k sides is the configuration obtained by:

(i) Dividing each side of a polygon into $n + 1$ equal parts by n points; and
(ii) Adding n line segments to join the corresponding pairs of such points on adjacent sides.
(iii)

3.1 Triangles in the n^{th} Subdivision

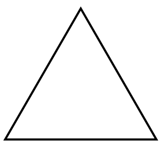
Theorem 3.1.1. Let $n,k \in \mathbb{N}$ with $n \geq 1$ and $k \geq 3$. Then the n^{th} subdivision of a regular polygon with k sides into triangles is given by the formula:

$$g_n(3,k) = \begin{cases} (n+1)^2, & k = 3 \\ kn, & k \geq 4 \end{cases}$$

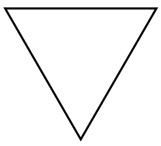
Proof. For K_3, the polygon is a triangle. Any triangle in the n^th subdivision is then form by 3 lines l_1,l_2 and l_3.

Any triangle in the n^{th} subdivision of a regular polygon is a triangle. Any triangle in the n^{th} subdivision is then formed by 3 lines l_1,l_2,l_3, as shown in the figure below.

Illustration 3.1.2: Two Possible Types of Triangles in the Subdivision



Type 1



Type 2

Suppose, l_2 is r units from the vertex A, we consider two case

Case 1: $r = n + 1$

When $r = n + 1$, then $l_1 = BC$. Consequently, the lines l_4 together with the intersecting lines l_2 and l_3 can only produce type 1 triangles.

Since there are $n + 1$ line segments of l_1 in the n^{th} subdivision and each line segment has a corresponding two-line segment (line segments from intersecting l_2 and l_3) who share the same end vertex, then the number of triangles for this case is also $n + 1$.

Case 2: $1 \leq r \leq n$

A similar argument as in Case 1 will show that there are r triangles of type 1 and r triangles of type 2 in the n^{th} subdivision. The number of type 1 and type 2
triangles are disjoint.

Total Number of Triangles

Therefore, the total number of triangles when $k = 3$ is given by:

$$g_n(3,3) = \sum_{r=1}^{n} 2r + (n+1)$$
$$= 2\sum_{r=1}^{n} r + (n+1)$$
$$= 2\frac{n(n+1)}{2} + (n+1)$$
$$= n(n+1) + (n+1) = (n+1)^2.$$

General Case for $k \geq 4$

Next, consider the polygon with sides $k \geq 4$. We claim that at least 2 vertices of the triangle in the n^{th} subdivision on the sides of the polygon. We prove this claim by contradiction.

Suppose to the contrary that no two vertices of the triangle $\{x,y,z\}$ in the n^{th} subdivision lie on the sides of the triangle. Then either the triangle contains exactly one vertex of the triangle that is not on the side of the polygon or no vertex of the triangle lies on the sides of the triangle.

Let $V_i, i = 1,2,...,k$ be the vertices of the polygon. Without loss of generality, we assume $V_j,V_{j+1}, j = 1,...,k - 1$ and V_k,V_1 are the sides of the polygon. By symmetry, we may consider the side V_1V_2 of the polygon.

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