*Proceeding International Conference on Religion, Science and Education* (2026) **5**, 1789-1799

# Batik Motifs from Mathematical Model Simulations of the Spread of Pulmonary Tuberculosis with the Use of Medical Masks

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**Abstract:** This study aims to create a mathematical model of the transmission of pulmonary tuberculosis where the transmission is prevented by medical masks. The model formulation is based on a system of differential equations that compartmentalizes the population into five groups, susceptible (S), infected (I), recovered (R), susceptible with medical masks (SM), and infected with medical masks (IM). The analytical steps taken yield a disease-free equilibrium point and an endemic equilibrium point with stability tested using the Jacobian matrix. Finally, as part of the numerical simulation of the model, real parameters of the city of Makassar were used to show that $R_0 < 1$, the disease will die out in the population; but if $R_0 > 1$ the disease will persist in the population, becoming endemic. As part of the novelty of this study, this model's simulated graphical results were turned into mathematical batik patterned art which tells a story about infection transmission and infection decline. The curves of the population and stability graphs were aesthetically interpreted into a motif that symbolizes these scientific findings to bridge the scientific world with cultured artistic sensibility. Therefore, this study provides scientific benefit to mathematical epidemiology and diversifies where scientific findings can be visualized into a teachable and significant local art piece.

**Keywords:** Tuberculosis, mathematical model, medical masks, numerical simulation, batik motif.

**Abbreviations:** TB – Tuberculosis; S - Susceptible (subpopulation); I - Infected (subpopulation); R - Recovered (subpopulation); SM - Susceptible subpopulation wearing medical masks; IM - Infected subpopulation wearing medical masks; $R_0$ - Basic reproduction number; DFE - Disease-Free Equilibrium; EE - Endemic Equilibrium; WHO - World Health Organization; HIV/AIDS - Human Immunodeficiency Virus/Acquired Immunodeficiency Syndrome; SIR - Susceptible-Infected-Recovered (model); UNESCO - United Nations Educational, Scientific and Cultural Organization; CR - Computed Radiography

## Introduction

Pulmonary tuberculosis remains one of the largest public health challenges in the world, wreaking havoc especially in developing nations. WHO epidemiological studies reveal that Mycobacterium tuberculosis - an obligate pathogen - causes pulmonary disease, although subsequent dissemination via extrapulmonary biopsies can occur. Transmission occurs via inhalation of infectious aerosols from the respiratory secretions of infected persons which means that such pathogens can become easily transmissible in crowded or poorly ventilated environments. Unfortunately, despite a host of diagnostic, therapeutic and preventative avenues available, tuberculosis continues to create high morbidity and mortality rates that strain international healthcare systems.

Indonesia is one of the world's top tuberculosis disease burden countries. Epidemiological research by the Indonesian Ministry of Health shows high TB incidence rates with new cases numbering in the thousands through active and passive case-finding annually. Within this highly burdensome nation, South Sulawesi Province is known to be one of the high-burden areas for tuberculosis. Various sociodemographic and structural factors


ISSN 2829-3355 | EISSN 2828-8467

contribute to this high burden - high population density, socioeconomic difficulties, limited healthcare access and low community awareness for preventative strategies. Such contextual factors call for an integrated approach to control featuring clinical intervention and community-based preventative programs.

Mathematical modeling serves as a highly reputable approach to assessing infectious disease transmission and spread. By using a compilation of differential equations based on compartment transitions appropriate for epidemiological study, researchers can retrospectively model patterns and prospectively project patterns of spread as well as effective interventions and policy development based on evidence. The Susceptible-Infected-Recovered (SIR) model and its derivatives have been derived in numerous ways in many infectious disease situations like the influenza epidemic, measles spread and HIV/AIDS transmission. Results from these models yield incredible findings pertaining to fundamental epidemiological parameters with the most important value being the basic reproduction number ($R_0$) - the threshold value that highlights potential for an outbreak should it be introduced to a susceptible population.

Medical mask usage has proven to be an effective non-pharmaceutical intervention to reduce pathogen spread in the air. Medical mask usage transformed into a large scale public health intervention during the global COVID-19 pandemic and was found to be effective in many epidemiological contexts to minimize transmission of infection. However, while mask usage is commonplace in medical settings to reduce occupational risk given exposure to tuberculosis, less mathematical modeling research of this kind has been done to ascertain mask use community wide. The ability to incorporate mask use variables into epidemiological models will allow researchers to not only estimate effectiveness of such options but also, customize the need for certain populations who would benefit from more access to masks versus those who could benefit from reduced access.

This research connects the fields of mathematical epidemiology and Indonesian batik production in a novel, interdisciplinary approach based on TB transmission patterns. Batik, a UNESCO Masterpiece of the Oral and Intangible Heritage of Humanity, relates to the textile art of Indonesia through patterned motifs with meaning. Thus, by converting patterns created by mathematical simulations into batik textiles, this research not only contributes to the overall scientific understanding of tuberculosis transmission, but also brings new science communication endeavors to a culturally interdisciplinary base. Motifs can be made for aesthetic purposes as well as for educational purposes that make the intricate mathematical concept accessible to various communities.

The purpose of this study is to create and validate a comprehensive mathematical model for pulmonary tuberculosis transmission with the intervention of medical mask usage as a controlling factor for prevention. Sub purposes include (1) finding the equilibrium points of the system, (2) assessing stability through a Jacobian Matrix, (3) determining the basic reproduction number, and (4) developing numerical simulations based on real life data taken from Makassar. Furthermore, it aims to validate how scientific visualization can become a culturally significant community and scholarly endeavor for greater knowledge dissemination efforts in the field of tuberculosis prevention.

## Mathematical Models

The mathematical framework of the study is based on a system of ordinary differential equations (ODEs) that describes the time evolution of transmission of pulmonary tuberculosis relative to proportions of mask wearing in the population. The total population was divided into five different compartments based on their epidemiological status and mask-wearing behavior, which are:

$S$ = Susceptible subpopulation

$I$ = Infected subpopulation

$R$ = Recovered subpopulation

$S_M$ = Susceptible subpopulation wearing medical masks

$I_M$ = Infected subpopulation wearing medical masks

The total population at any time $t$ is given by:

$$N = S + I + R + S_M + I_M$$

The model assumes the following parameters:
$\mu$ = Natural birth/death rate
$\beta$ = Rate of contact between infected subpopulations and tuberculosis bacteria
$\rho$ = Recovery rate of subpopulations after infection
$N$ = Population size
$\gamma$ = Level of awareness among subpopulations regarding the use of medical masks

Based on these assumptions, the dynamics of the system can be expressed by the following set of differential equations:

$$\frac{dS}{dt} = \mu N + \gamma_2 S_M - \mu S - \gamma_1 S - \beta SI$$

$$\frac{dI}{dt} = \beta SI + \gamma_4 I_M - \gamma_1 I - \rho I - \mu I$$

$$\frac{dS_M}{dt} = \gamma_1 S - \gamma_2 S_M - \mu S_M$$

$$\frac{dI_M}{dt} = \gamma_1 I + \gamma_3 I_M - \mu I_M - \rho I_M$$

$$\frac{dR}{dt} = \rho I + \rho I_M - \mu R$$

**Equilibrium Point**

**Disease-Free** ($i = 0$ *and* $i_M = 0$)

$$\mu + \gamma_2 s_M - \mu s - \gamma_1 s = 0 \quad ..(1)$$

$$\beta s \cdot 0 + \gamma_4 \cdot 0 - \gamma_1 \cdot 0 - \rho \cdot 0 - \mu \cdot 0 = 0 \quad ..(2)$$

$$\gamma_1 s - \gamma_2 s_M - \mu s_M = 0 \quad ..(3)$$

$$\gamma_1 \cdot 0 + \gamma_3 \cdot 0 - \mu \cdot 0 - \rho \cdot 0 = 0 \quad ..(4)$$

$$\gamma_1 s - \gamma_2 s_M - \mu s_M = 0$$

$$\gamma_1 s = (\gamma_2 + \mu) s_M$$

$$s_M = \frac{\gamma_1 s}{\gamma_2 + \mu}$$

Substitute $s_M$ into equation (2)

$$\mu + \gamma_2 \left(\frac{\gamma_1 s}{\gamma_2 + \mu}\right) - \mu s - \gamma_1 s = 0$$

$$\mu + \frac{\gamma_2 \gamma_1 s}{\gamma_2 + \mu} - \mu s - \gamma_1 s = 0$$

$$\mu(\gamma_2 + \mu) + \gamma_2 \gamma_1 s - \mu s(\gamma_2 + \mu) - \gamma_1 s(\gamma_2 + \mu) = 0$$

$$\mu\gamma_2 + \mu^2 + \gamma_2\gamma_1 s - \mu s\gamma_2 - \mu^2 s - \gamma_1 s\gamma_2 - \gamma_1 s\mu = 0$$

$$(\gamma_2\gamma_1 - \mu\gamma_2 - \mu\gamma_1 - \mu^2)s + \mu\gamma_2 + \mu^2 = 0$$

$$s(\gamma_2\gamma_1 - \mu\gamma_2 - \mu\gamma_1 - \mu^2) = -\mu\gamma_2 - \mu^2$$

$$s = \frac{-\mu\gamma_2 - \mu^2}{\gamma_2\gamma_1 - \mu\gamma_2 - \mu\gamma_1 - \mu^2}$$

$s + s_M = 1 - i - i_M - r$ and because $i = i_M = 0$ and r irrelevant then $s + s_M = 1$. Substitute $s_M = \frac{\gamma_1 s}{\gamma_2 + \mu}$ into $s + s_M = 1$:

$$s + \frac{\gamma_1 s}{\gamma_2 + \mu} = 1$$

$$s\left(1 + \frac{\gamma_1}{\gamma_2 + \mu}\right) = 1$$

$$s\left(\frac{\gamma_2 + \mu + \gamma_1}{\gamma_2 + \mu}\right) = 1$$

$$s = \frac{\gamma_2 + \mu}{\gamma_2 + \mu + \gamma_1}$$

$$s_M = \frac{\gamma_1 s}{\gamma_2 + \mu} = \frac{\gamma_1 \cdot \frac{\gamma_2 + \mu}{\gamma_2 + \mu + \gamma_1}}{\gamma_2 + \mu} = \frac{\gamma_1}{\gamma_2 + \mu + \gamma_1}$$

Thus, the point of disease-free equilibrium

$$E_0 = \left(\frac{\mu + \gamma_2}{\mu + \gamma_1 + \gamma_2}, 0, \frac{\gamma_1}{\mu + \gamma_1 + \gamma_2}, 0\right)$$

At this point, the system equilibrates as long as birth, death, and change of behavior with the masks operate in equilibrium. The first compartment is the susceptibility level without a mask; it is the sensitivity/rate out of the total transition rate. The second compartment is the fraction of susceptibility, i.e., healthy individuals who wear masks (the larger part percentage in an overall equation). In a sense, this equilibrium means the population is aware of this preventative measure and acts accordingly. Thus, equilibrium and stability can only exist under the constraint of $R_0 < 1$- in other words, that on average, each infected person can infect less than one other person, so the disease dies out through the population.

**Affected by Tuberculosis**

$$\frac{ds}{dt} = \mu - \beta si - \beta si_M + \gamma_2 s_M - \gamma_1 s - \mu s$$

$$\frac{di}{dt} = \beta si + \beta si_M + \gamma_4 i_M - \gamma_3 i - \rho i - \mu i$$

$$\frac{ds_M}{dt} = \gamma_1 s - \gamma_2 s_M - \mu s_M$$

$$\frac{di_M}{dt} = \gamma_3 i - \gamma_4 i_M - \rho i_M - \mu i_M$$

with $s + i + r + s_M + i_M = 1$

For the endemic point E*= $s*, i*, s_M*, i_M*$, set each derivative equal to zero:

$$\mu - \beta s^* i^* - \beta s^* i_M^* + \gamma_2 s_M^* - \gamma_1 s^* - \mu s^* = 0 \quad (1)$$

$$\beta s^* i^* + \beta s^* i_M^* + \gamma_4 i_M^* - \gamma_3 i^* - \rho i^* - \mu i^* = 0 \quad (2)$$

$$\gamma_1 s^* - \gamma_2 s_M^* - \mu s_M^* = 0 \quad (3)$$

$$\gamma_3 i^* - \gamma_4 i_M^* - \rho i_M^* - \mu i_M^* = 0 \quad (4)$$

From equation (3):

$$\gamma_1 s^* - \gamma_2 s_M^* - \mu s_M^* = 0$$

$$\gamma_1 s^* = (\gamma_2 + \mu) s_M^*$$

$$s_M^* = \frac{\gamma_1 s^*}{\gamma_2 + \mu}$$

From equation (4):

$$\gamma_3 i^* - \gamma_4 i_M^* - \rho i_M^* - \mu i_M^* = 0$$

$$\gamma_3 i^* = (\gamma_4 + \rho + \mu) i_M^*$$

$$i_M^* = \frac{\gamma_3 i^*}{\gamma_4 + \rho + \mu}$$

Assume that $E = \gamma_2 + \mu$ and $D = \gamma_4 + \rho + \mu$

Substitute $i_M*$ into equation (2):

$$\beta s^* i^* + \beta s^* \left(\frac{\gamma_3 i^*}{D}\right) + \gamma_4 \left(\frac{\gamma_3 i^*}{D}\right) - \gamma_3 i^* - \rho i^* - \mu i^* = 0$$

$$i^* \left[\beta s^* + \beta s^* \frac{\gamma_3}{D} + \gamma_4 \frac{\gamma_3}{D} - \gamma_3 - \rho - \mu\right] = 0$$

$$\beta s^* \left(1 + \frac{\gamma_3}{D}\right) + \frac{\gamma_3 \gamma_4}{D} = \gamma_3 + \rho + \mu$$

$$\beta s^* \left(1 + \frac{\gamma_3}{D}\right) = \gamma_3 + \rho + \mu - \frac{\gamma_3 \gamma_4}{D}$$

$$s^* = \frac{\gamma_3 + \rho + \mu - \frac{\gamma_3 \gamma_4}{D}}{\beta \left(1 + \frac{\gamma_3}{D}\right)}$$

$$s^* = \frac{\rho + \mu}{\beta}$$

Substitute s*, $s_M$*, and $i_M$* into equation (1):

$$\mu - \beta\left(\frac{\rho + \mu}{\beta}\right) i^* - \beta\left(\frac{\rho + \mu}{\beta}\right) i_M^* + \gamma_2\left(\frac{\gamma_1}{\gamma_2 + \mu} \cdot \frac{\rho + \mu}{\beta}\right) - \gamma_1\left(\frac{\rho + \mu}{\beta}\right) - \mu\left(\frac{\rho + \mu}{\beta}\right) = 0$$

$$i^* = \frac{\mu\left[\begin{matrix}\beta(\gamma_2\gamma_4 + \gamma_2\mu + \gamma_2\rho + \gamma_4\mu + \mu^2 + \mu\rho) - \gamma_1(\gamma_4\mu + \gamma_4\rho + \mu^2 + 2\mu\rho + \rho^2) - \\ \gamma_2(\gamma_4\mu + \gamma_4\rho + \mu^2 + 2\mu\rho + \rho^2) - \gamma_4\mu^2 - \gamma_4\mu\rho - \mu^3 - 2\mu^2\rho - \mu\rho^2\end{matrix}\right]}{\beta\left[\begin{matrix}\gamma_2\gamma_3\mu + \gamma_2\gamma_3\rho + \gamma_2\gamma_4\mu + \gamma_2\gamma_4\rho + \gamma_2\mu^2 + 2\gamma_2\mu\rho + \gamma_2\rho^2 + \gamma_3\mu^2 + \\ \gamma_3\mu\rho + \gamma_4\mu^2 + \gamma_4\mu\rho + \mu^3 + 2\mu^2\rho + \mu\rho^2\end{matrix}\right]}$$

Subtitute s* and i* into $s_M^*$ and $i_M^*$

$$s_M^* = \frac{\gamma_1(\rho + \mu)}{\beta(\gamma_2 + \mu)}$$

$$i_M^* = \frac{\gamma_3 i^*}{\gamma_4 + \rho + \mu}$$

So, the equilibrium point is affected by tuberculosis. ($S^*$, $I^*$, $S_M^*$, $I_M^*$):

$$E^* = \left(\frac{\rho + \mu}{\beta}, i^*, \frac{\gamma_1(\rho + \mu)}{\beta(\gamma_2 + \mu)}, \frac{\gamma_3 i^*}{\gamma_4 + \rho + \mu}\right)$$

The endemic equilibrium is the amount of tuberculosis that is consistently found in the population over time. The fractional proportion of susceptible individuals who keeps this endemic equilibrium (s∗) equals the diseased group per individual recovered (ϱ) or per individual dead (μ) relative to the transmission parameter. Biologically, this suggests that the greater the transmission parameter, the lesser the fractional proportion of susceptibles needed to sustain transmission to sustain infection. However, the higher the recovery or death parameters, the greater the susceptibles need to be, in order to maintain the infection cycle. Endemic equilibrium exists when $R_0 < 1$, meaning that those infected are projected to spread the illness to 1+ other people; thus, it can remain within the community population without any additional external forces.

**Existence**

$\mu, \gamma_1, \gamma_2 > 0$ and $\beta(\gamma_2 + \mu) > (\gamma_1 + \gamma_2 + \mu)(\mu + \rho)$

The values $\mu, \gamma_1, \gamma_2 > 0$ tell the researcher that natural birth/death (μ), mask adoption rate ($\gamma_1$), mask release rate ($\gamma_2$) must all be greater than 0, meaning, in a realistic situation, there will always be a natural propensity to be born/die as well as a natural propensity to begin/stop wearing a mask. Furthermore, the inequality $\beta(\gamma_2+\mu) > (\gamma_1+\gamma_2+\mu)(\mu+\rho)$ is the condition for the endemic equilibrium point to exist. This can be interpreted that the force of transmission of the disease (β) multiplied by the transitional force from wearing a mask to non-mask wearing ($\gamma_2+\mu$) is greater than the transitional forces resulting from recovery (ϱ) and with natural mortality (μ) compounded by a mindset shift ($\gamma_1+\gamma_2+\mu$). In other words, the only way for the infection to exist and thrive within the population is if its force of transmission is stronger than those who adapt a new mindset of recovery and death without infection complications. If this is not true enough, then the infection cannot sustain itself and will plummet more and more until it disappears from the population. Naturally, this demonstrates that TB transmission will become endemic when avenues of infectious interaction and less motivation in regards to mask wearing proves greater than natural means of recovery and death. Thus, it also shows how more can be done to promote better community understanding of mask use and quicker recovery methods to deter such probable potential for future existence.

**Disease-Free Point Stability**

$$J(E_i) = \begin{pmatrix} -\gamma_1-\mu & -\beta s_0 & \gamma_2 & -\beta s_0 \\ 0 & \beta s_0 - r_3 - \mu - \rho & 0 & \beta s_0 + \gamma_4 \\ \gamma_1 & 0 & -\gamma_2-\mu & 0 \\ 0 & \gamma_3 & 0 & -\gamma_4-\rho-\mu \end{pmatrix}$$

$$\frac{ds}{dt} = \mu + \gamma_2 s_M - \mu s - \gamma_1 s - \beta SI$$

$$\frac{di}{dt} = \beta si + \gamma_4 i_M - \gamma_1 i - \rho i - \mu i$$

$$\frac{ds_M}{dt} = \gamma_1 s - \gamma_2 s_M - \mu s_M$$

$$\frac{di_M}{dt} = \gamma_1 i + \gamma_3 i_M - \mu i_M - \rho i_M$$

The jacobian matrix

$$J = \begin{pmatrix} \frac{\partial\left(\frac{ds}{dt}\right)}{\partial s} & \frac{\partial\left(\frac{ds}{dt}\right)}{\partial i} & \frac{\partial\left(\frac{ds}{dt}\right)}{\partial s_M} & \frac{\partial\left(\frac{ds}{dt}\right)}{\partial i_M} \\ \frac{\partial\left(\frac{di}{dt}\right)}{\partial s} & \frac{\partial\left(\frac{di}{dt}\right)}{\partial i} & \frac{\partial\left(\frac{di}{dt}\right)}{\partial s_M} & \frac{\partial\left(\frac{di}{dt}\right)}{\partial i_M} \\ \frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial s} & \frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial i} & \frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial s_M} & \frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial i_M} \\ \frac{\partial\left(\frac{di_M}{dt}\right)}{\partial s} & \frac{\partial\left(\frac{di_M}{dt}\right)}{\partial i} & \frac{\partial\left(\frac{di_M}{dt}\right)}{\partial s_M} & \frac{\partial\left(\frac{di_M}{dt}\right)}{\partial i_M} \end{pmatrix}$$

For the equation $\frac{ds}{dt} = \mu + \gamma_2 s_M - \mu s - \gamma_1 s - \beta SI$

- $\frac{\partial\left(\frac{ds}{dt}\right)}{\partial s} = -\mu - \gamma_1 - \beta i$
- $\frac{\partial\left(\frac{ds}{dt}\right)}{\partial i} = -\beta s$
- $\frac{\partial\left(\frac{ds}{dt}\right)}{\partial s_M} = \gamma_2$
- $\frac{\partial\left(\frac{ds}{dt}\right)}{\partial i_M} = 0$

For the equation $\frac{di}{dt} = \beta si + \gamma_4 i_M - \gamma_1 i - \rho i - \mu i$

- $\frac{\partial\left(\frac{di}{dt}\right)}{\partial s} = \beta i$
- $\frac{\partial\left(\frac{di}{dt}\right)}{\partial i} = \beta s - \gamma_3 - \rho - \mu$
- $\frac{\partial\left(\frac{di}{dt}\right)}{\partial s_M} = 0$
- $\frac{\partial\left(\frac{di}{dt}\right)}{\partial i_M} = \gamma_4$

For the equation $\frac{ds_M}{dt} = \gamma_1 s - \gamma_2 s_M - \mu s_M$

- $\frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial s} = \gamma_1$
- $\frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial i} = 0$
- $\frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial s_M} = -\gamma_2 - \mu$

- $\frac{\partial\left(\frac{ds_M}{dt}\right)}{\partial i_M} = 0$

  For the equation $\frac{di_M}{dt} = \gamma_1 i + \gamma_3 i_M - \mu i_M - \rho i_M$

- $\frac{\partial\left(\frac{di_M}{dt}\right)}{\partial s} = 0$
- $\frac{\partial\left(\frac{di_M}{dt}\right)}{\partial i} = \gamma_3$
- $\frac{\partial\left(\frac{di_M}{dt}\right)}{\partial s_M} = 0$
- $\frac{\partial\left(\frac{di_M}{dt}\right)}{\partial i_M} = -\gamma_4 - \rho - \mu$

Then the Jacobian matrix is

$$\begin{pmatrix} -\mu-\gamma_1-\beta_i & -\beta s & \gamma_2 & 0 \\ \beta_i & \beta_2-\gamma_3-\rho-\mu & 0 & \gamma_4 \\ \gamma_1 & 0 & -\gamma_2-\mu & 0 \\ 0 & \gamma_3 & 0 & -\gamma_4-\rho-\mu \end{pmatrix}$$

Each entry in the matrix represents how small increases in one population compartment dictates the change in another. For example, in the first row alone, the diagonal entry $(-\mu-\gamma_1-\beta_i)$ where i equals the infected dynamic system, represents the rate at which individuals leave the non-mask susceptible class and the off-diagonal entries represent the driven change between compartments. For example, $-\beta s$ represents the loss of susceptible population as a driven dynamic system from others who are actively transmitting infection, while $\gamma_2$ represents the increase from individuals who are taking masks off and joining back into the susceptible population. Ultimately, eigenvalue analysis determines if a dynamic system will want to return to equilibrium (stable) or move further away (unstable). Therefore, knowing whether this dynamic system is stable or unstable provides insight into whether public health measures impact disease transmission in the long term and if it is effective and/or practical in the real world.

**Model Simulation**

A tuberculosis (TB) transmission model simulation focusing on interpersonal interactions was conducted using a series of parameters collected from various research studies and applied to the geographical context of Makassar City. This city is the capital of South Sulawesi Province, Indonesia, with an area of 175.77 km, and is geographically located on the southwestern coast of Sulawesi Island. This model uses population data of 1,449,401 people in 2015 to calculate and determine all the parameter values needed to analyze the dynamics of TB in this urban environment.

Table 1. Numerical simulation parameter values for the spread of tuberculosis in Makassar City

| Parameter | Value |
|---|---|
| $\boldsymbol{\mu}$ | $4{,}7442791 \times 10^{-2}$ |
| $\boldsymbol{\beta}$ | $6{,}32570545 \times 10^{-4}$ |
| $\boldsymbol{\rho}$ | $5{,}5556 \times 10^{-3}$ |
| $\boldsymbol{\gamma_1}$ | $2{,}3 \times 10^{-2}$ |
| $\boldsymbol{\gamma_2}$ | $6{,}6 \times 10^{-8}$ |
| $\boldsymbol{\gamma_3}$ | $7 \times 10^{-2}$ |
| $\boldsymbol{\gamma_4}$ | $1{,}3 \times 10^{-8}$ |

Simulation results using the Mathlab program and based on the parameters in Table 1 and with initial values $s(0) = 0{,}3; i(0) = 0{,}2; s_M(0) = 0{,}1; i_M(0) = 0{,}2$ obtained and presented in the following figure.

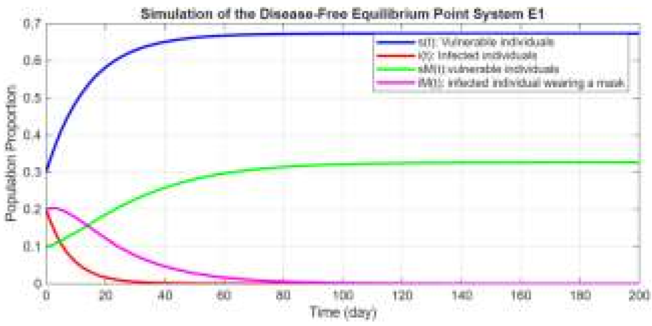

Figure 1. Simulation of a disease-free equilibrium point system $E_1$

Figure 1 shows that the susceptible population without medical masks decreased to 0 (stable) on day 50 and remained stable. The susceptible population with medical masks increased (stable) on day 100 at 0.3. The infected population without medical masks decreased to 0 (stable) on day 100 and remained stable. The infected population with medical masks decreased to 0 (stable) on day 100 and remained stable.

The results from this simulation show that if $R_0 < 1$, then the disease dies out in the population following the 50th day. This supports Theorem 2, which has already been proven. This is a relevant, yet not significantly proven, random occurrence because when susceptibility and infection rates are low enough, awareness of mask wearing should decrease $R_0$.

Next, for the simulation $R_0 > 1$. With the parameter $\beta$ increased by $2 \times 10^{-3}$.

$$\beta = 6{,}32570545 \times 10^{-4} \times 2000$$
$$= 6{,}32570545 \times 10^{-1}$$

With initial values $s(0) = 0{,}3; i(0) = 0{,}2; s_M(0) = 0{,}1; i_M(0) = 0{,}2$ we obtain $\mathcal{R}_0 = 0{,}002357476323$ and presented in the following figure.

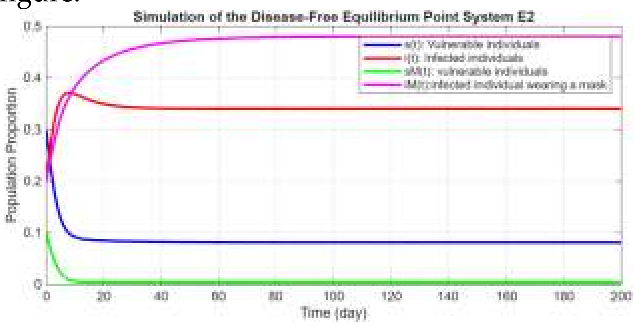

Figure 2. Simulation of a disease-free equilibrium point system $E_2$

In Figure 2, The proportion of susceptible (wearing a mask) started off increasing but started to drop after day ten where by day 100 the final equation was 0.33 and had stabilized. The proportion of infected (not wearing a mask) increased steadily until day 100 where by that day the final equation was 0.58 and stabilized. The susceptible population (wearing a mask) decreased from the beginning where after day 15 the same population continued to decrease until it stabilized at a value of 0,002 on day 75. The infected (not wearing a mask) decreased through this time and stabilized at a value of 0,0005 around day 50 and stabilized thereafter.

We see from the simulation that $R_0 < 1$ results in elimination of the disease from the population after roughly 50 days and $R_0 > 1$ results in the disease persisting, even with masks. This is proof that masks and management of the value of $R_0$ are critical to reducing the incidence of disease.

## Batik Motifs

The combination of mathematical discoveries with batik designs through creative methods creates an original connection between scientific knowledge and cultural traditions. The model output trajectories between susceptible and infected populations form the basis for creating visual patterns in this research. The population changes through time become visible through these curves which show how numbers expand before reaching stability.

The infection curve motif consists of connected lines which acknowledge how TB spreads by means of community connection, this idea that less and less people get infected on the infection curve results in an undulating pattern symbolic of recovery and community herd immunity. Designers use symmetrical patterns because this ideal is found by means of medical studies and batik style theory across the board.

The use of color in this design provides specific meanings. The darkness of red and brown hues is symbolic of the tenacity and fight of the disease, however, the shift to blue and greens symbolize health recovery and community herd immunity. The geometric shapes that were formed by the intersections of the mathematical equations during the simulation are findings that represent the stability and harmony of these mathematical systems and similar systems in Javanese ideology.

Where mathematics becomes pictorially representative of a human saga and teamwork is with this motif. This final product shows the connection between scientific findings and artistic creations with educational value and cultural value. It brings new bridges of understanding to light that allow students to learn mathematical principles by conceptualizing their application in a culturally relevant way.


Figure 3. Tubercolosis batik motif

**The Biological and Philosophical Meaning of Batik Motifs**

The flowers, leaves, and vines of this batik motif grow together in a soft, undulating motion pattern. Each component of this batik looks alive, moving with one another, each with beauty that exists not just on a surface level, but an intrinsic, biological level as well. Biologically the flowers and leaves symbolize re-birth from growing, fertilizing, subsequent growing seasons. The twining creeper of the vine motif symbolizes the way in which plants grasp light (phototropism) symbolizes the adaptive qualities of living things as they grasp for survival through their environments.

From an ecological aspect, this batik displays an ecosystem equilibrium that creates the foundation for sustainability on earth. Green occurs in dominant frequency as the color of vegetation and fertility, blue is the cooling component of water, while red is the energy and strength of sunlight. Yet when combined, these three colors operate symbiotically supported by visually aesthetic patterns like biological connections found in the earth, water and energy that together symbolize the three core components that underlie any component's existence. Thus, the visual symbolism between the three colors indicates that life cannot exist independently but instead must exist symbiotically and interactively, in a wholeness.

In addition, the various floral shaped patterns and symbols on this batik represent bio-diversity. In ecological patterns, bio-diversity is a positive resource that maintains ecosystem equilibrium. From a philosophical perspective, every shape and every color play an important role like every biological unit maintains equilibrium in nature. Therefore, this pattern works to symbolize not only the beautiful elements of nature but also an environmentally conscious one since humans are a part of nature and must maintain biodiversity for things to flourish. This is also extended to the shapes of the various patterns that are nice looking, thus, humans must appreciate things outside themselves for life to work.

From a philosophical perspective, this symbol has a very deep meaning relative to humans and the universe. The curved vines represent the path of human life - full of struggles but eventually rising toward a greater intention. Thus, the subtle waviness to it symbolized surrender, wisdom as well as the ability for man to become open-minded to change. From a Javanese perspective, given that there are many curved lines, these also symbolize laku - the path of life taken with patience, sincerity and a peaceful mind.

The flowers in bloom on this batik motif represent perfection and beauty through a long process. It symbolizes that every human being is a flower that blossoms at the right time, having gone through a growing process. Furthermore, the light colors of the petals give an impression of softness and sincerity while the dark background supports determination and power, and the combination of both symbolizes the beauty and fighter's attribute of living one's life.

Furthermore, from the perspective of religion, this batik motif symbolizes microcosm and macrocosm. The repetition of the motif represents the order of the natural environment and the universal principles of life that define existence - and everything in the world has its own order. Therefore, when looking at things from an Eastern perspective, the order represents God's will which forces any matter to have balance in life. Thus, wearing and/or making this type of batik means honoring the balance of natural matter and God's praise.

Thus, from a general perspective, biological and philosophical meaning comes together as one major message: "The harmony of life." This Batik works reminds humans to live harmoniously within nature, balance everything out and appreciate differences. From each blooming flower to each curving leaf to each curling line - everything speaks of life, beauty and wisdom that comes from this batik. This is not just a work of beauty but a reflection on works in nature to maintain a spiritual connection between macrocosm and microcosm to support that life will be beautiful as long as we are able to tread harmoniously with nature and balance everything out.

## Conclusions

Therefore, the transmission aspect of TB is patterned scientifically, culturally and humanely through a batik motif. For the deviation from the research pattern, however, the female and male aspects of this pattern can be wearing masks (a preventative control measure) to combat transmission. The DFE created exists as locally asymptotically stable for $R_0 < 1$ which states that in a population without increasing transmission (waving masks or ineffective mask usage), tuberculosis can cease transmission. But should $R_0 > 1$, equilibrium exists - as a steady state exists (and will forever exist), for those people who carry it will always make it a viable option to remain within population confines. The more effective mask use (as people wear them in other situations as well) indicates people are less likely to catch it so prevalence decreases over time. Thus, it does not matter how long TB has existed as endemic (as preventative, precautionary measures with wearing masks), this airborne contagious disease public health concern can be reduced to such low levels it's far more manageable.

Yet beyond the biological reality, the researchers create a batik motif from what they observe from their simulations to match the reality of scientific, culturally and humanistically survival approach. The consistent crest and trough overlap with infection and recovery over time - and therefore, through theme - the distance and amplitude of the two patterns become smaller. Symbolically, this means an increased power and natural tendency for prevention as these crest heights become smaller. Philosophically this means a balance of susceptibility and empathy to provide relief. If people don't care about prevention - because they are ignorant to awareness or disinterest in discipline or empathetic consideration for others - then there will be no batik motif - but with empathy comes unity, making public health considerations a cultured motif.

Ultimately this study makes interdisciplinary connection to science that finds biological themes within it all and emerges culturally more integrated. The motif batik title of TB can have a beautiful name and reality behind it by linking information that would otherwise be confined strictly within regions where life can coincide and connect fostering awareness and human survival.

**Acknowledgements:** The authors thank the Mathematics Department at Faculty of Science and Technology UIN Sunan Kalijaga Yogyakarta for their academic and technical assistance during this research. The authors thank all colleagues and students who shared their ideas and participated in discussions about modeling and batik design development. The authors thank the Makassar Health Department for sharing essential epidemiological information which supported this research. The authors thank to Batik Ghakhuka, Address: Mredo, Bangunharjo, Sewon, Bantul, Special Region of Yogyakarta, Indonesia, which is used as a place for batik making.

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